Webinar
Hypothesis Testing

Math 201
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What do you think?

- The Framulator car company claims their new 2019 F2000 sedans average 50mpg for its hybrid vehicle fleet
  - Your sample of 100 vehicles gives average of 47
  - Is that enough evidence to discard their claim?
  - Can we believe Framulator or not?

Hypothesis Tests

- Definition:
  - Uses *sample* statistics
  - Tests claim made about a *population* parameter
- Example
  - Framulator "average 50mpg for its hybrid fleet"
  - Your sample of 100 vehicles gives average of 47
  - Is that enough evidence to discard their claim?

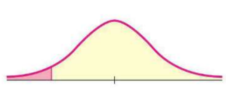
Stating Your Hypothesis

- Null hypothesis H_0
  - Contains statement of equality $H_0 = k$
- Alternative hypothesis H_1
  - Contains statement of strict *in*equality $<$ or $>$ or $\neq$
  - Which one used will depend on the situation
- If you suspect the actual value is higher/lower
  - Then $H_1 > k$ or $H_1 < k$
- If you just think it's different
  - Then $H_1 \neq k$

Types of Tests

TYPES OF STATISTICAL TESTS

A statistical test is:
- **left-tailed** if H_1 states that the parameter is less than the value claimed in H_0.
- **right-tailed** if H_1 states that the parameter is greater than the value claimed in H_0.
- **two-tailed** if H_1 states that the parameter is different from (or not equal to) the value claimed in H_0.



Stating Your Hypothesis

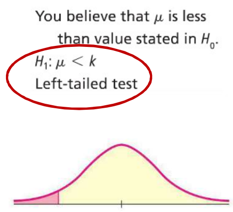
- Null hypothesis

Claim about μ or historical value of μ

$H_0: \mu = k$

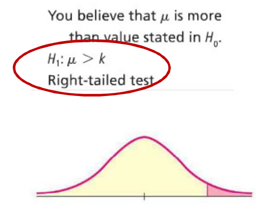
Stating Your Hypothesis

- Alternate hypothesis and type of test



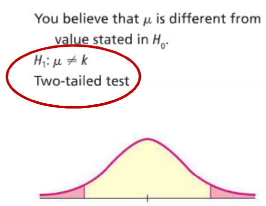
Stating Your Hypothesis

- Alternate hypothesis and type of test



Stating Your Hypothesis

- Alternate hypothesis and type of test

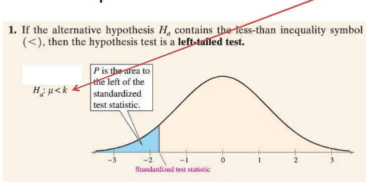


Level of Significance

- Definition: the *level of significance*
  - Max allowable probability of making type 1 error
  - Denoted by α
- Problems typically specify an α value
  - Used to look up value in table
  - That value compared to a computed value
  - Determines whether to reject or fail to reject H_0

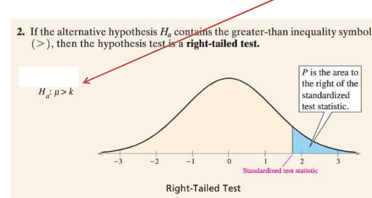
Types of Tests

- Left tailed test
  - Determined by direction of H_1 inequality <
  - You believe μ is less than value stated in H_0



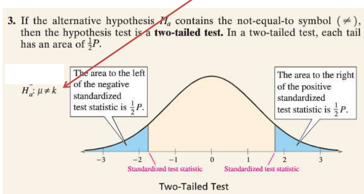
Types of Tests

- Right tailed test
  - Determined by direction of H_1 inequality >
  - You believe μ is more than value stated in H_0



Types of Tests

- Two tailed test
  - Determined by H_1 not equal sign $\neq$
  - You believe μ is different from value stated in H_0



Making & Interpreting Decision

- Failing to reject H_0
  - NOT the same as *accepting* H_0
  - NOT the same as H_0 = TRUE
  - Means *not enough evidence* to reject H_0

Making & Interpreting Decision

- Refer to this table for help in interpreting

	Claim	
Decision	**Conclusion about H_0**	**Conclusion about H_1**
Reject H_0.	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Types of Errors

- Type 1 Error
  - H_0 is rejected … but it is TRUE
- Type 2 Error
  - H_0 is NOT rejected … but it is FALSE

	Truth of H_0	
Decision	**H_0 is true.**	**H_0 is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Types of Statistical Tests

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 8.2 σ known), t (Section 8.2 σ unknown)
p	$\hat{p}$	z (Section 8.3)
σ^2	s^2	χ^2 (Section 10.1 - 10.3)

- We will be given
  - Parameter for the population
  - Test statistic (for subset of total population)
- We will calculate a standardized test statistic
  - Use to compare to specified α

Online Videos

F.Y.I.

- Your Cengage account provides *excellent* videos
- Make sure to view them in addition to viewing this presentation
- Seeing/hearing two different explanations will
  - Further your understanding
  - Give different viewpoints

Online Videos

F.Y.I.

- In your assignments view on WebAssign

WEEK 5 INTRO: WATCH VIDEOS — view
Week 5: Exercises (6.4 - 7.3) — view
WEEK 6 INTRO: WATCH VIDEOS — view
Week 6: Exercises (8.1-8.3) — view
Exam 3 Review (6.4 - 8.3) — view
Exam 3 (6.4 - 8.3) — view

Click on the "view" link.

Steps for Hypothesis Testing

- Homeowners claim μ speed for cars on their street is > 35 mph speed limit
  - Random sample of 100 cars has $\bar{x}$ = 36
  - Given σ = 4
  - Support claim at α = 0.05

Here we will use the z-Test for the mean when σ is known.

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

H_0: ? H_1 : ?

Steps for Hypothesis Testing

- Homeowners claim μ speed for cars on their street is > 35 mph speed limit

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

H_0: ? H_1 : ?

- H_0 : μ = 35
- H_1 : μ > 35

a) $\mu \le 35$
b) $\mu > 35$
c) $\mu \ge 35$
d) $\mu < 35$
e) $\mu = 35$

Respond in chat box

Steps for Hypothesis Testing

- Home owners claim μ speed for cars on their street is > 35 mph posted limit
  - Random sample of 100 cars has $\bar{x} = 36$
  - Given σ = 4
  - Support claim at α = 0.05

H_0: ? H_a: ?

2. Specify the level of significance.

α = ?

Steps for Hypothesis Testing

- H_0 : μ = 35 H_1 : μ > 35
- Which test do we want ?
  - a) Left tailed test
  - b) Right tailed test
  - c) Two tailed test
  - d) Ring tailed test

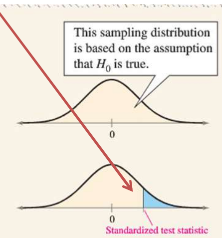
Steps for Hypothesis Testing

- H_0 : μ = 35 H_1 : μ > 35

3. Determine the standardized sampling distribution and sketch its graph.

Use α with tables or with technology

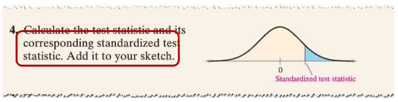
4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.



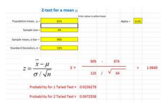
Steps for Hypothesis Testing

- Identify parameters needed for the formula
  - $\bar{x} \neq 36$, n = 100, μ = 35, σ = 4

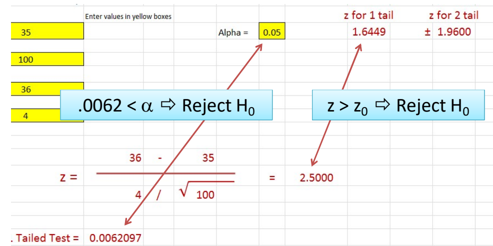
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



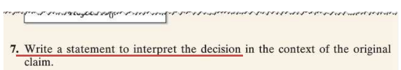
- Use Excel Tool



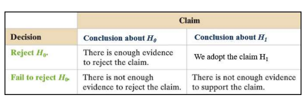
Steps for Hypothesis Testing



Steps for Hypothesis Testing



- Recall $H_0 : \mu = 35$ $H_1 : \mu > 35$ We have rejected H_0
- There is enough evidence
  - At the 5% level
  - To adopt the claim
  - $H_1 : \mu > 35$



Decision	Claim: Conclusion about H_0	Claim: Conclusion about H_1
Reject H_0	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

When You Know μ, But *not* σ

- Use t-Test $t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$
  - Check t-value against values in table 6
- Insurance agent says μ cost to insure < $1200
  - Random sample of 7 similar quotes $\bar{x}$ = $1125
  - Standard deviation for sample, s = $55
  - Is there enough evidence to support agent's claim at α = 0.10

When You Know μ, But *not* σ

- Insurance agent says μ cost to insure < $1200
  - Random sample of 7 similar quotes $\bar{x}$ = $1125
  - Standard deviation for sample, s = $55
  - Is there enough evidence to support agent's claim at α = 0.10
- Hypotheses

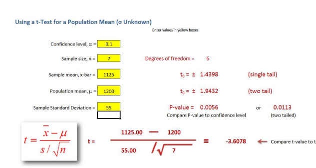
H_0 : μ = $1200
H_1 : μ < $1200

T-Test – We know μ, Not σ

- Identify parameters needed for the formula

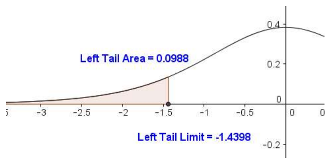
$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

- $\bar{x}$ = 1125 μ = 1200 s = 55 n = 7
- Use Excel tool



T-Test – We know μ, Not σ

- Recall: H_0: μ = 1200 H_1: μ < 1200
- Inequality for H_1 tells us *left tail test*



We must check : t < -1.4398 ?

T-Test – We know μ, Not σ

- Recall: H_0: μ = 1200 H_1: μ < 1200 (claim)

Using a t-Test for a Population Mean (σ Unknown)

Enter values in yellow boxes

Confidence level, α =	0.1				
Sample size, n =	7	Degrees of freedom =	6		
Sample mean, x-bar =	1125	t_0 = ±	1.4398	(single tail)	
Population mean, μ =	1200	t_0 = ±	1.9432	(two tail)	
Sample Standard Deviation =	55	P-value =	0.0056	or	0.0113
		Compare P-value to confidence level			(two tailed)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

t = (1125.00 − 1200) / (55.00 / √7) = -3.6078 ← Compare t-value to t_0

- So, what do we conclude?
  a) Accept H_0 c) Accept H_1
  b) Reject H_0 d) Reject H_1

T-Test – We know μ, Not σ

- Recall: H_0: μ = 1200 H_1: μ < 1200 (claim)

t_0 = ± 1.4398 (single tail)
t_0 = ± 1.9432 (two tail)
P-value = 0.0056 or 0.0113
Compare P-value to confidence level (two tailed)

t = (1125.00 − 1200) / (55.00 / √7) = -3.6078 ← Compare t-value to t_0

- Note: -3.6078 < -1.4398 ⇨ Reject H_0

T-Test – We know μ, Not σ

- Recall: H_0: μ ≥ 1200 H_1 : μ < 1200 (claim)
- We have rejected H_0

	Claim	
Decision	**Conclusion about H_0**	**Conclusion about H_1**
Reject H_0.	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

- Interpretation:
  There is enough evidence
  - At the 10% level of significance
  - To adopt the insurance agent's claim
  - That the mean cost of insurance for this category of cars is less than $1200

Using P-Values with t-Test

- P-values have to do with *probability*
  - Of a test statistic (such as the t-value)
  - Compared to the specified α value
- Note results in Excel tool

Less than α = 0.10 in our example
So, reject H_0

t_0 = ± 1.4398 (single tail)
t_0 = ± 1.9432 (two tail)
P-value = 0.0056 or 0.0113
Compare P-value to confidence level (two tailed)

t = (1125.00 − 1200) / (55.00 / √7) = -3.6078 ← Compare t-value to t_0

Using P-Values with t-Test

- P-values have to do with *probability*
  - Of a test statistic (such as the t-value)
  - Compared to the specified α value
- Another Excel Tool … finds p-value only

	A	B	C	D	E	F
1		Finding p-value for a t-distribution				
2		Enter values in yellow boxes				
3				Right tail		
4	Test stastistic =	1.863		p-value =	0.03764	
5						
6	Degrees of freedom =	23		Left tail		
7				p-value =	0.96236	
8						
9				Two tail		
10				p-value =	0.07528	

Hypothesis Test for Proportions

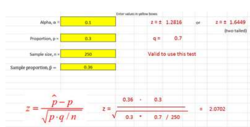
- This is a z-test
- Calculate z with formula $z = \dfrac{\hat{p} - p}{\sqrt{p \cdot q / n}}$
- $\hat{p}$ is the sample statistic, the proportion for the subset of the population
- Requirements
  - $n \cdot p \geq 5$ and $n \cdot q \geq 5$

Hypothesis Test for Proportions

- Researcher claims 30% of U.S adults turned off by distasteful ads of given product
  - You do random sample of 250 adults
  - 90 say yes, they are turned off by distasteful ad
  - At $\alpha = 0.10$, is there enough evidence to reject claim?
- Hypotheses: $H_0: p = .30$ (claim) $H_1: p \neq .30$

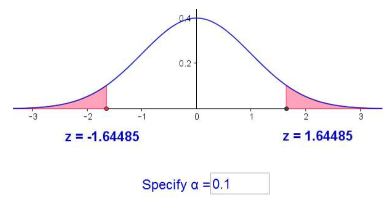
Hypothesis Test for Proportions

- Check if this will be a valid test (p =.3, q = .7)
  - $n \cdot p = 250 \cdot 0.3 = 75 > 5$
  - $n \cdot q = 250 \cdot 0.7 = 175 > 5$
- Go to Excel tool for this test



Hypothesis Test for Proportions

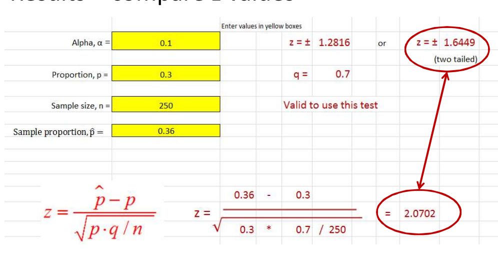
- $H_1: p \neq 0.3$ tells us *two tailed test*



- Use tables or technology to get z-values

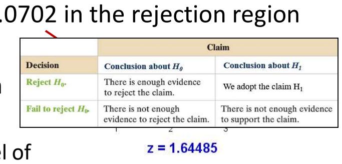
Hypothesis Test for Proportions

- Results – compare z values



Hypothesis Test for Proportions

- Recall $H_0: p = .30$ (claim) $H_1: p \neq .30$
- We found z = 2.0702 in the rejection region
  - Reject H_0
- There is enough evidence
  - at the 10% level of significance
  - To reject the claim that 30% are turned off by distasteful ad



	Claim	
Decision	Conclusion about H_0	Conclusion about H_1
Reject H_0	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Hypothesis Test for Variance, Standard Deviation

- Company claims σ < 1.4 min for incoming call to reach correct office
  - Random sample of 25 calls
  - s = 1.1
  - At α = 0.10, is there enough evidence to support company's claim?
- Hypotheses
  - H_0 : σ = 1.4 H_1 : σ < 1.4 (claim)

Hypothesis Test for Variance, Standard Deviation

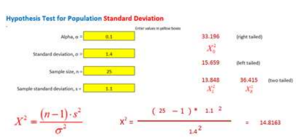
- This is a Chi-Square test
- Calculate with formula $X^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$
- Where
  - σ^2 = population statistic
  - s^2 = statistic for subset of population
  - n = size of subset (n – 1) = degrees of freedom for use in tables

Hypothesis Test for Variance, Standard Deviation

- Identify values for formula
  - s = 1.1
  - n = 25 d.f. = 24
  - σ = 1.4

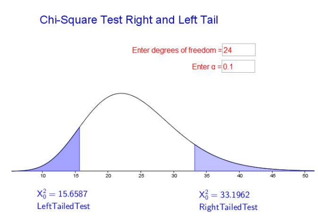
$$X^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$$

- Use Excel Tool



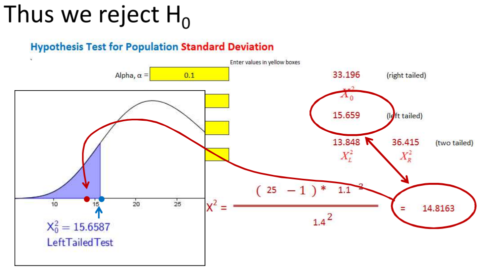
Hypothesis Test for Variance, Standard Deviation

- Recall: H_0 : σ = 1.4 H_1 : σ < 1.4 (claim)
- Left Tailed X^2 test



Hypothesis Test for Variance, Standard Deviation

- Results – note $X^2 < X_0^2$ ... in rejection area
- Thus we reject H_0



Hypothesis Test for Variance, Standard Deviation

- Recall: H_0 : σ = 1.4 H_1 : σ < 1.4 (claim)
- We have rejected H_0
- Thus we conclude there is enough evidence
  - At the 10% level of significance
  - To support the claim
  - The standard deviation for time for incoming call is less than 1.4 minutes

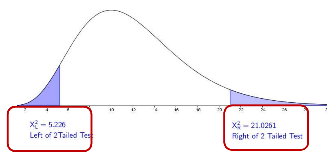
	Claim	
Decision	**Conclusion about H_0**	**Conclusion about H_1**
Reject H_0.	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Hypothesis Test for Variance, Standard Deviation

- The previous problem dealt with standard deviation … let's try one for *variance* …
- A diet product company claims variance of weight loss of their users = 25.5 (assume normal distribution)
  - Random sample of 13 users
  - Variance for sample = 10.8
  - At α = 0.10, determine if enough evidence to reject company's claim

Hypothesis Test for Variance, Standard Deviation

- Identify hypotheses & claim
  - $H_0 : \sigma^2 = 25.5$ (claim) $H_1 : \sigma^2 \neq 25.5$
- Note level of significance, degrees of freedom
  - $\alpha = 0.10$, d.f. = n – 1 = 13 – 1 = 12
- Note this is a two tailed test, determine X_L^2 and X_R^2

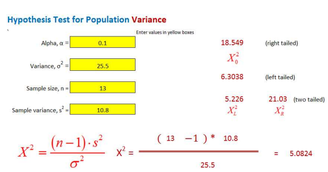


Hypothesis Test for Variance, Standard Deviation

- Determine standardized test statistic using formula

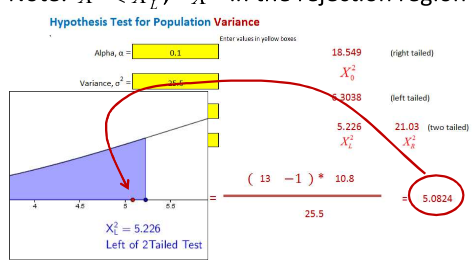
$$X^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$$

- Use Excel Tool



Hypothesis Test for Variance, Standard Deviation

- Note: $X^2 < X_L^2$, X^2 in the rejection region



Hypothesis Test for Variance, Standard Deviation

- Interpretation
- Recall $H_0 : \sigma^2 = 25.5$ (claim) $H_1 : \sigma^2 \neq 25.5$
- We have rejected H_0
- We determine there is enough evidence
  - At 10% level of significance
  - To reject company's *claim* (variance of weight losses is 25.5)

	Claim	
Decision	**Conclusion about H_0**	**Conclusion about H_1**
Reject H_0.	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Review: Types of Statistical Tests

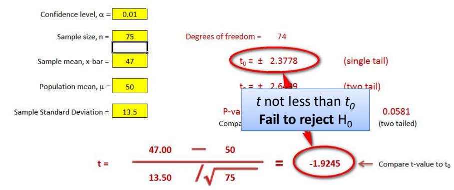
Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 8.2 σ known), t (Section 8.2 σ unknown)
p	$\hat{p}$	z (Section 8.3)
σ^2	s^2	χ^2 (Section 10.1 - 10.3)

- We were given
  - Parameter for the population
  - Test statistic (for subset of total population)
- We will calculate a standardized test statistic
  - Use to compare to specified α

What do you think?

- The Framulator car company claims their 2019 F2000 sedans average is at least 50mpg for its hybrid vehicle fleet
- Your sample of 100 vehicles gives average of 47 with sample sd = 13.5
- Let $\alpha = 0.01$
- $H_0 : \mu = 50$ (Claim)
- $H_1 : \mu < 50$

Result



Result

- Fail to reject H_0: $\mu \geq 50$ (Claim)
-

	Claim	
Decision	**Conclusion about H_0**	**Conclusion about H_1**
Reject H_0.	There is enough evidence to reject the claim.	We adopt the claim H_1
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

- Our evidence is insufficient!

Aids to Calculations

- Excel tools
  - z-Test for μ, σ is known
  - t-Test for μ, σ is *not* known
  - z-Test for *p* (proportion)
  - X^2 –Test for standard deviation
  - X^2 –Test for variance
- Download these from Dr. Armstrong's Web
  http://www.biblestudiesbysteve.com/HypothesisTesting/

Aids to Calculations

- Links to Geogebra Apps
  - Left and right z-Test critical values
    http://ggbtu.be/mlhQkUm9N
  - Two tail z-Test critical values
    http://ggbtu.be/mBrWQFKnk
  - Left and right t-Test critical values
    http://ggbtu.be/mkUcdX6RW
  - Two tail t-Test critical values
    http://ggbtu.be/mtlcI7o06
  - Left and right tail Chi-square critical values
    http://ggbtu.be/m582043
  - Two tail Chi-square critical values
    http://ggbtu.be/mJzCPGPBE