Using Normal Probability Distributions

Webinar Slides

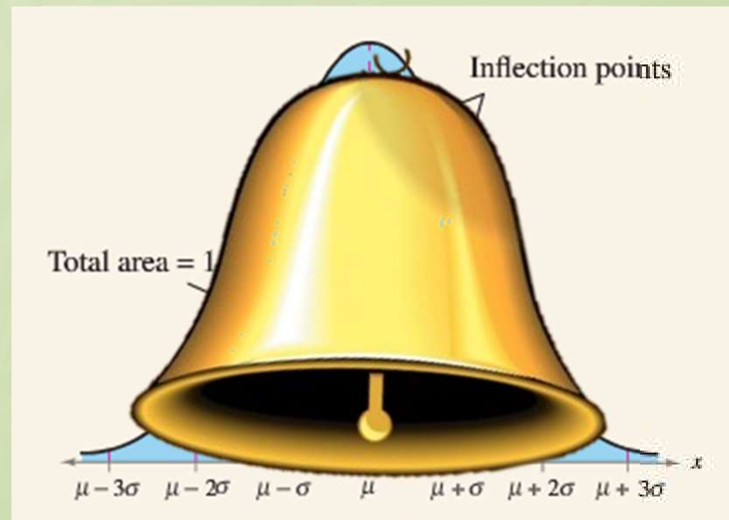
Remember when …

- What did you think when a teacher told said she/he had “graded on the curve”?
- Typical questions from my students
  - “Did you curve the test?”
  - “Was there mercy and grace?”
  - “Did you add some sugar to the scores?
  - “What if we all flunked?”

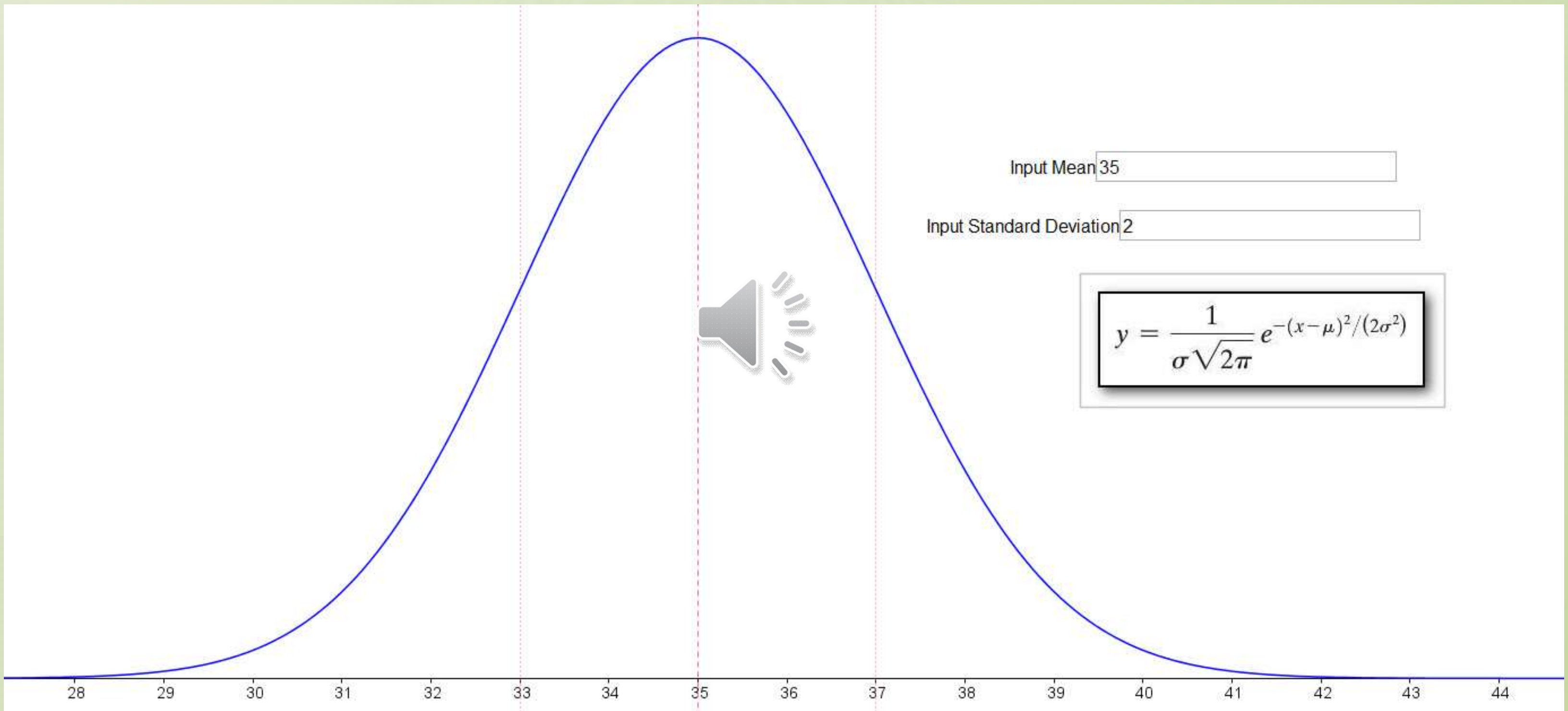
CONCEPTS AND PRASE

Properties of a Normal Distribution

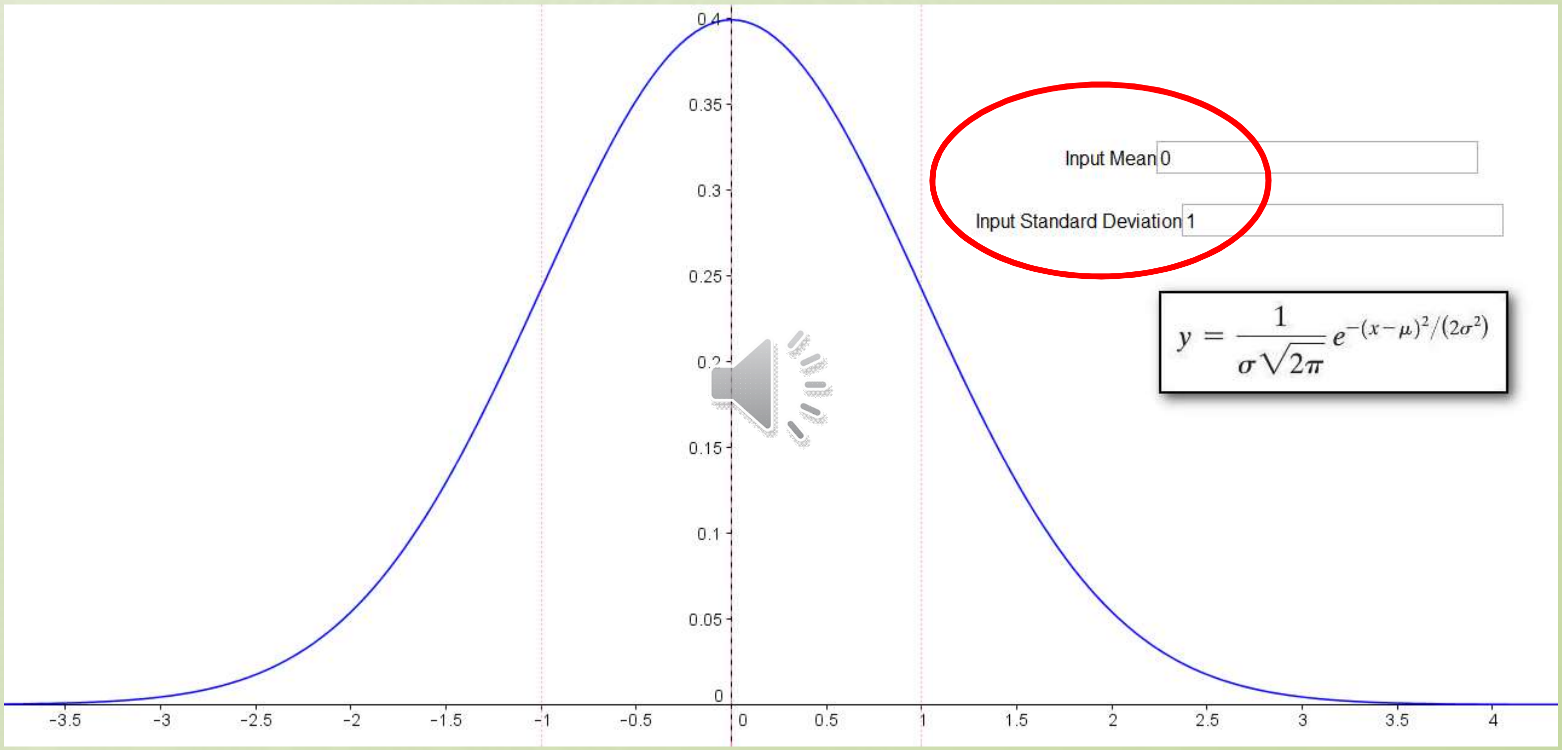
- Mean, median, and mode are equal.
- Normal curve bell-shaped, symmetric about mean.
- Total area under normal curve is equal to 1.
- Normal curve approaches, but never touches, x-axis
- Inflection points at $\pm 1\ \sigma$



Normal Curve



Standard Normal Curve



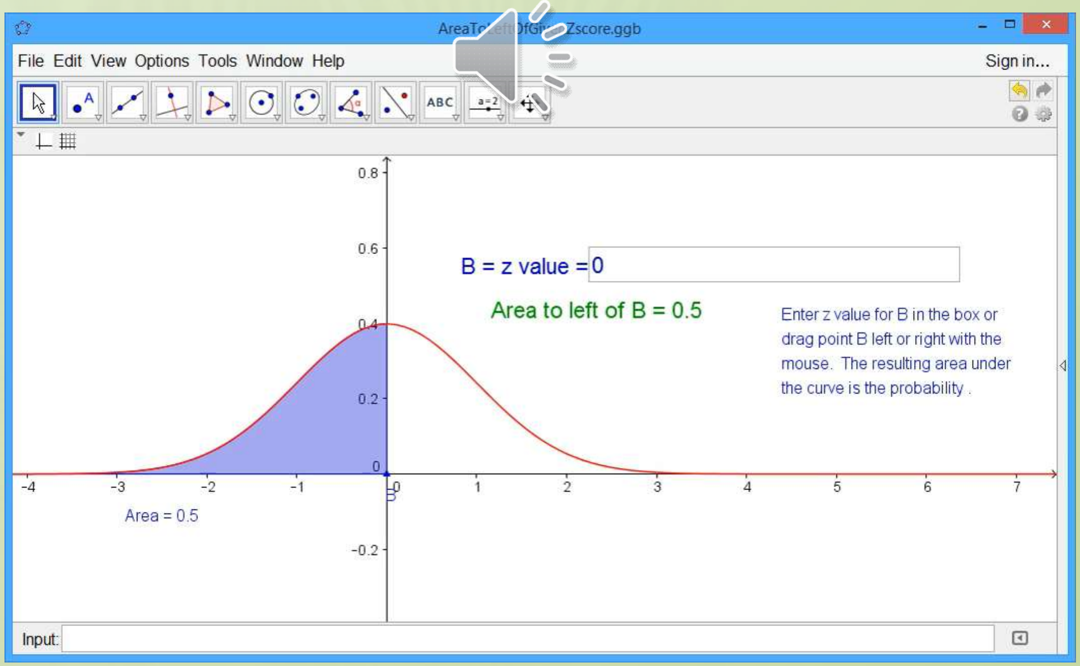
$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

Total area under the curve = 1

Standard Normal Distribution

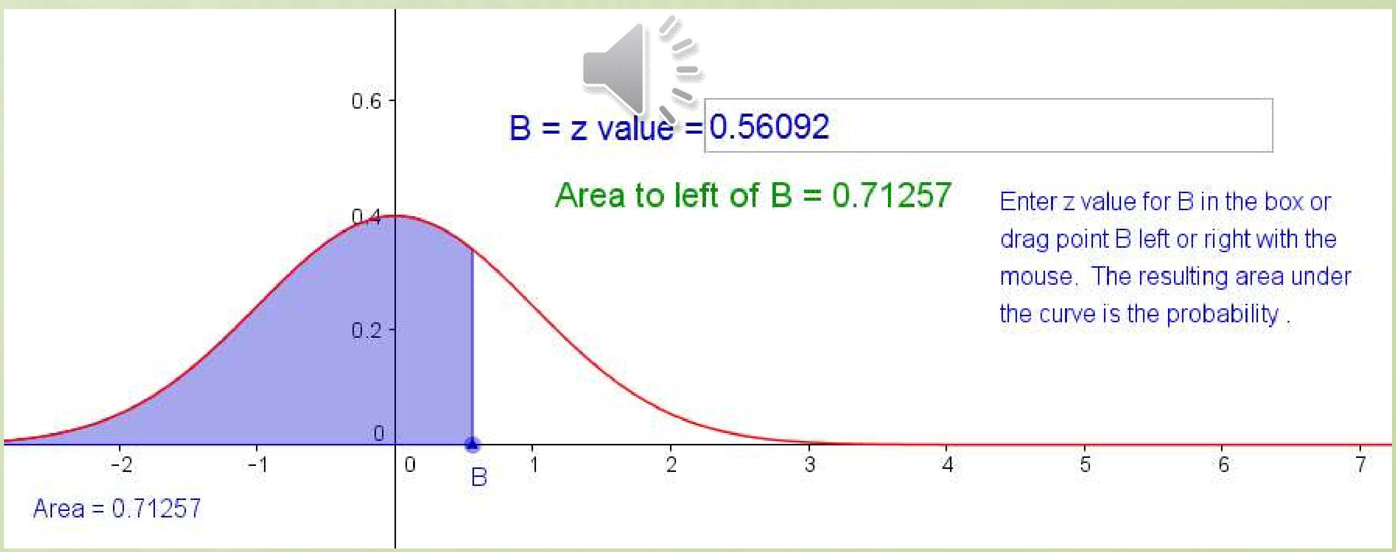
PROPERTIES OF THE STANDARD NORMAL DISTRIBUTION

1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.
2. The cumulative area increases as the z-scores increase.
3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for z-scores close to $z = 3.49$.



Standard Normal Curve

- You can access this program at https://www.geogebra.org/m/B2cLwp5y



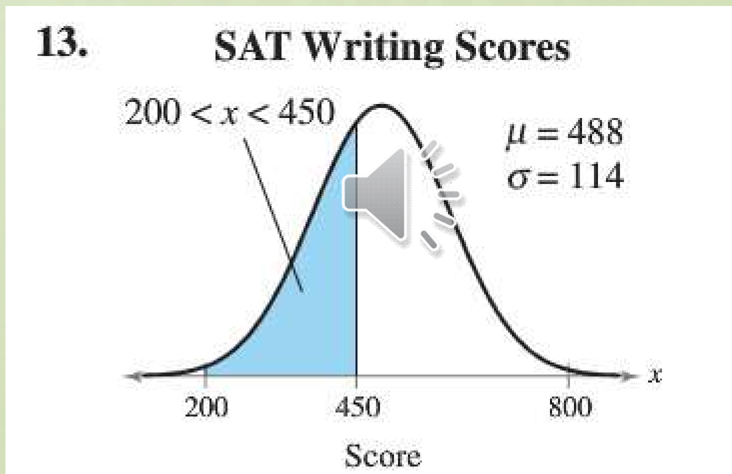
Standard Normal Curve

- If you've taken any calculus, what's going on here? What calculus process are we doing to find the area under the curve?

$$\int_a^b f(x)\ dx$$

Try It Out …

- Consider this problem



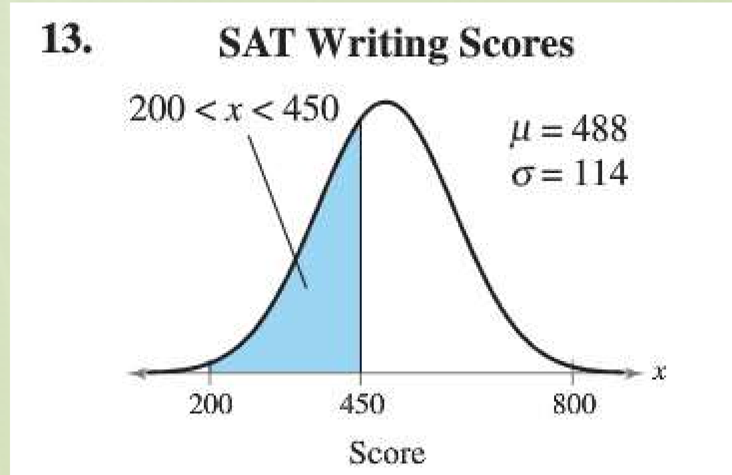
- Find the probability of a score falling between the two given values.

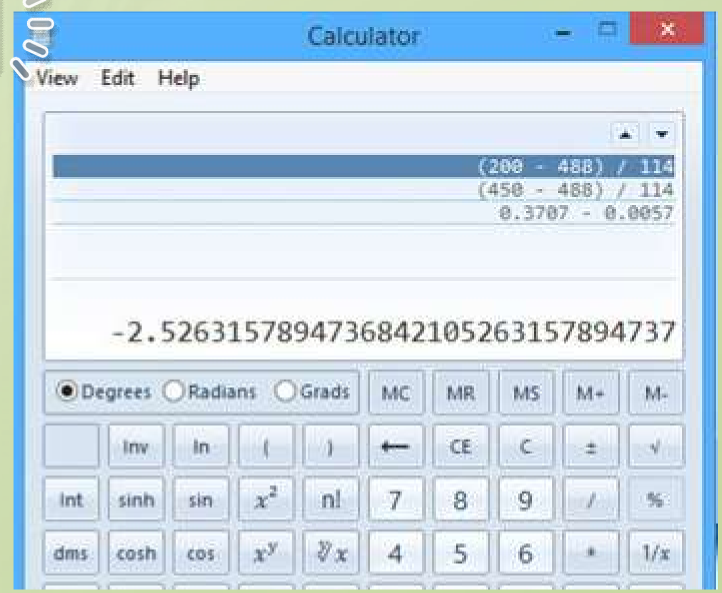
Try It Out

- We know

$$z = \frac{x - \mu}{\sigma}$$

- Calculate z-score for 200

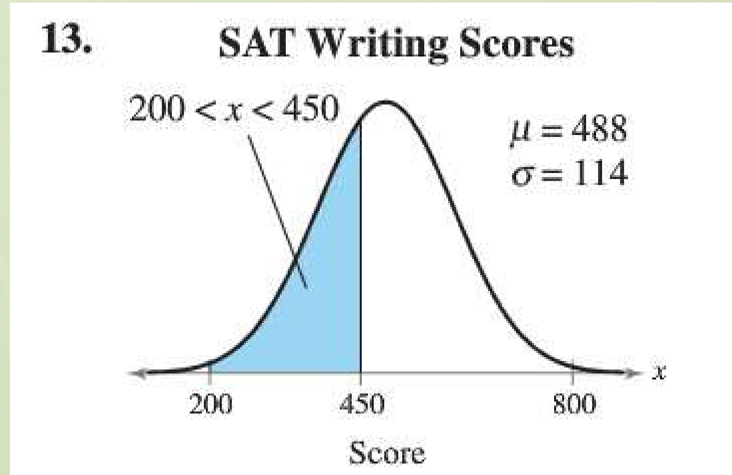




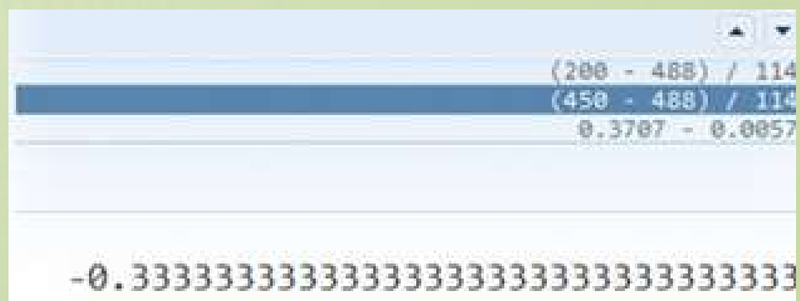
Try It Out

- We know

$$z = \frac{x - \mu}{\sigma}$$



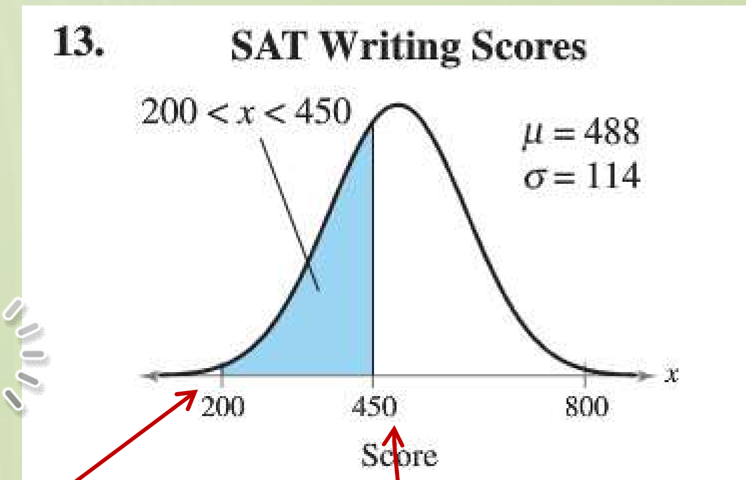
- Calculate z-score for 200
- And for 450



Try It Out …

- We know

$$z = \frac{x - \mu}{\sigma}$$



- z-score for 200

z = -2.526

- And for 450

z = -0.333

Try It Out …

- Now look up values in Table 5

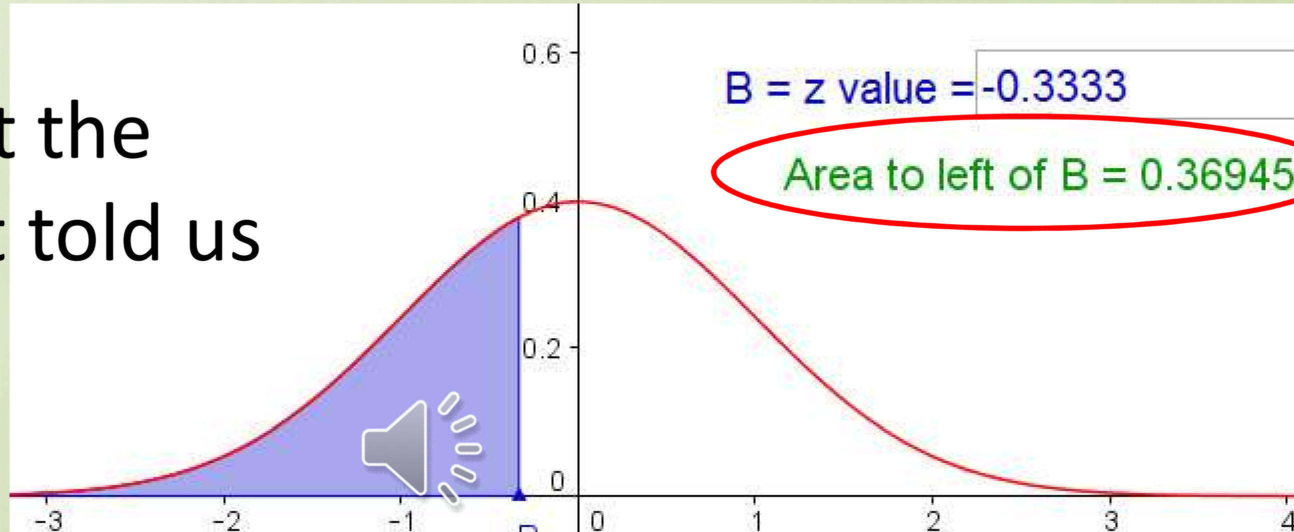
TABLE 5 Areas of a Standard Normal Distribution

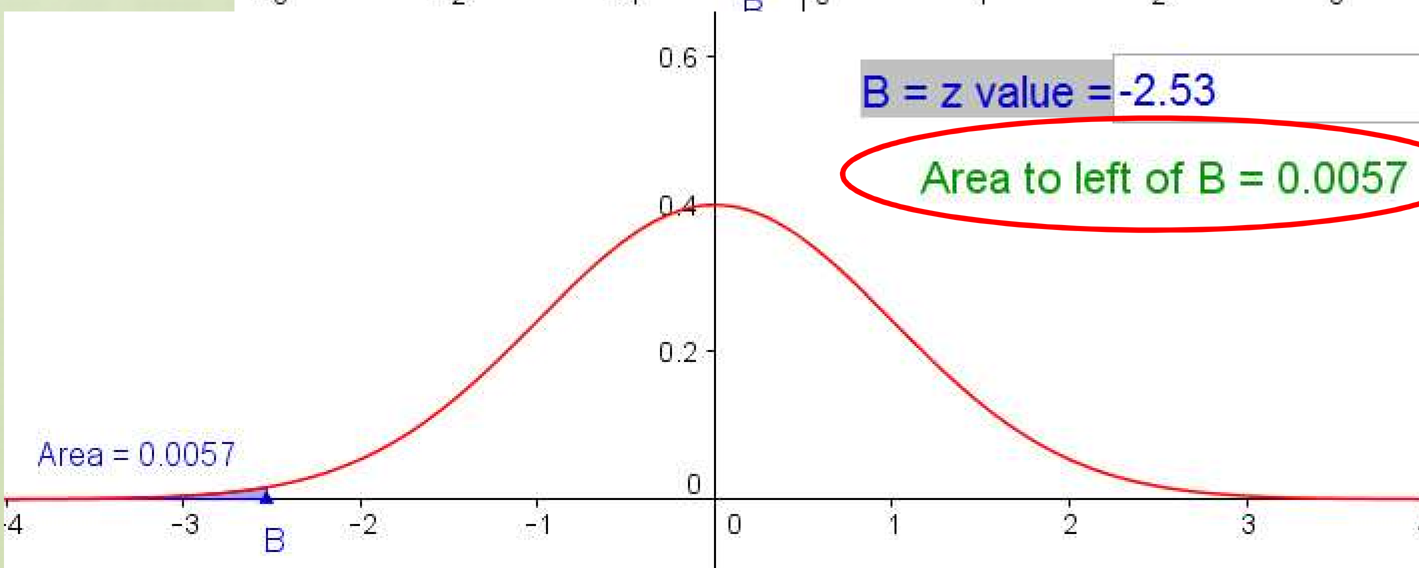
z	.09	.08	.07	.06	.05	.04	.03	.02
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018
−0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358
−0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107

Try It Out …

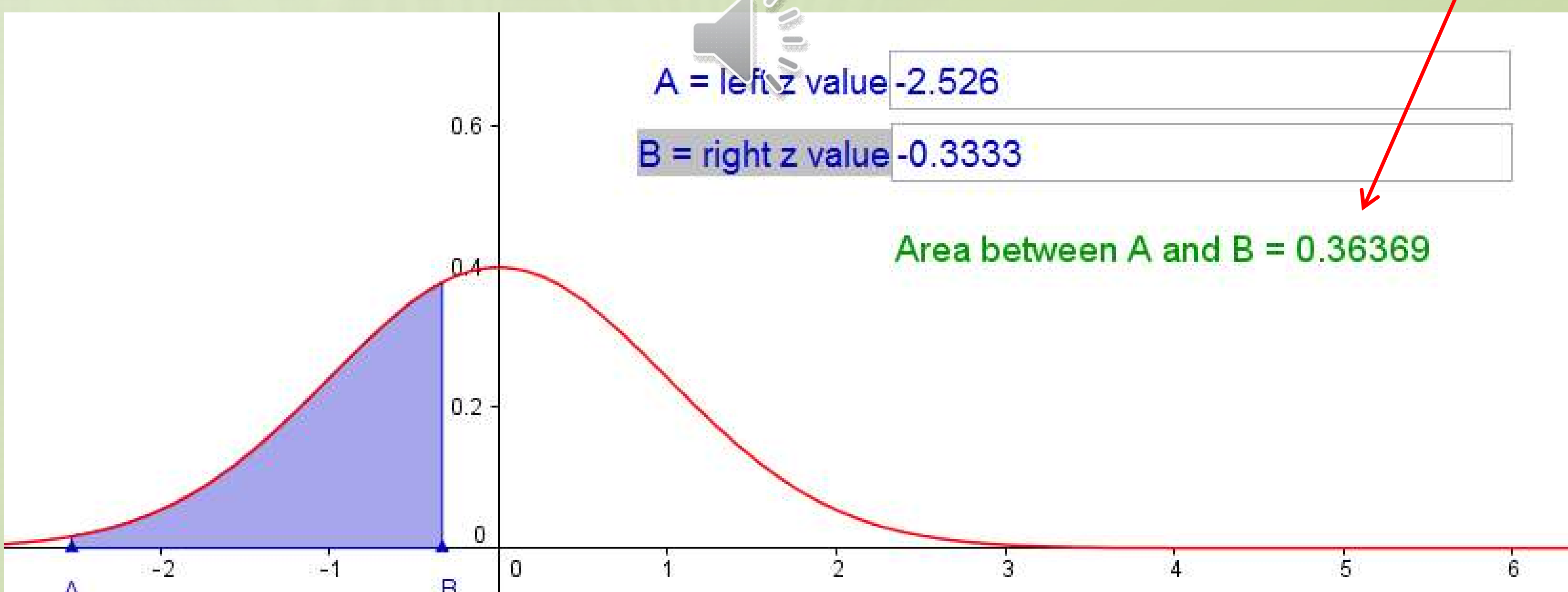
- Note what the tables just told us





Try It Out …

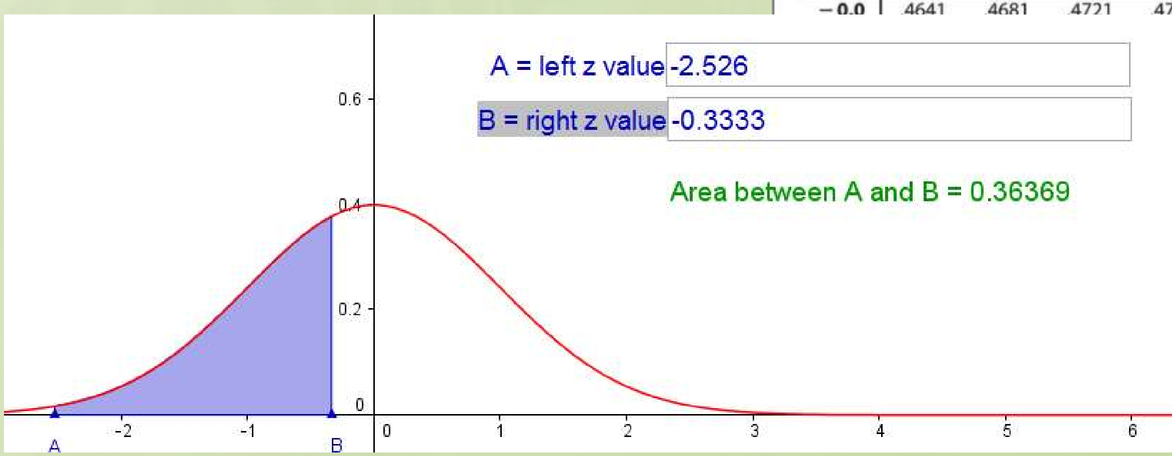
- Now we subtract
  to get area between …



Why the difference

- Why does the app and the tables give different values?

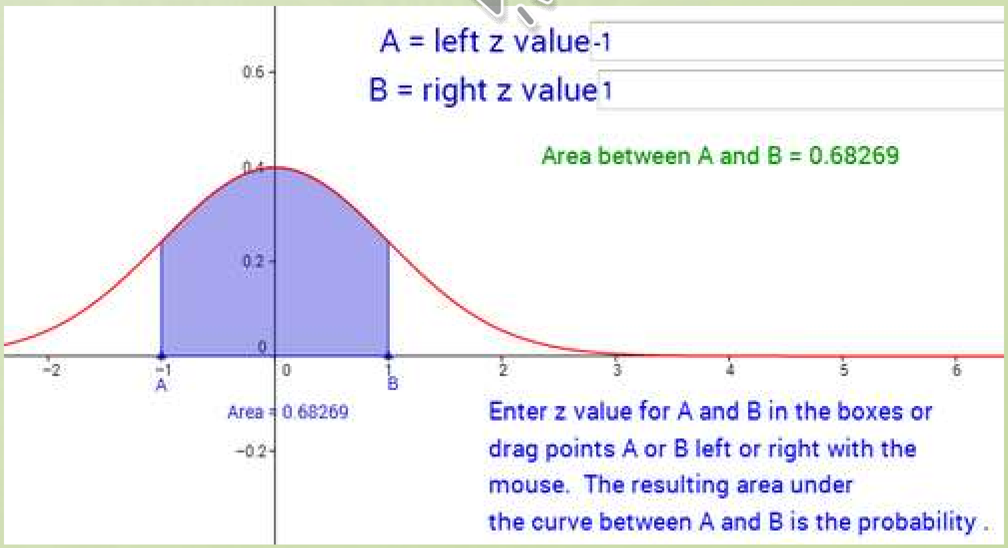
z	.09	.08	.07	.06	.05	.04	.03	.02
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018
−0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358
−0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920



Another Version

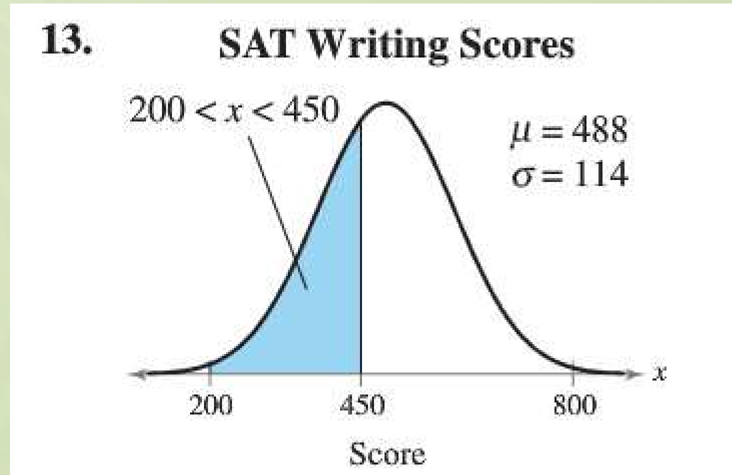
- This program is similar … also available to you
  - Does much of the work for you

https://www.geogebra.org/m/URLUI9OZ



Use Technology

- Excel can also do this easily
- The probability of a score less than between 200 and 450

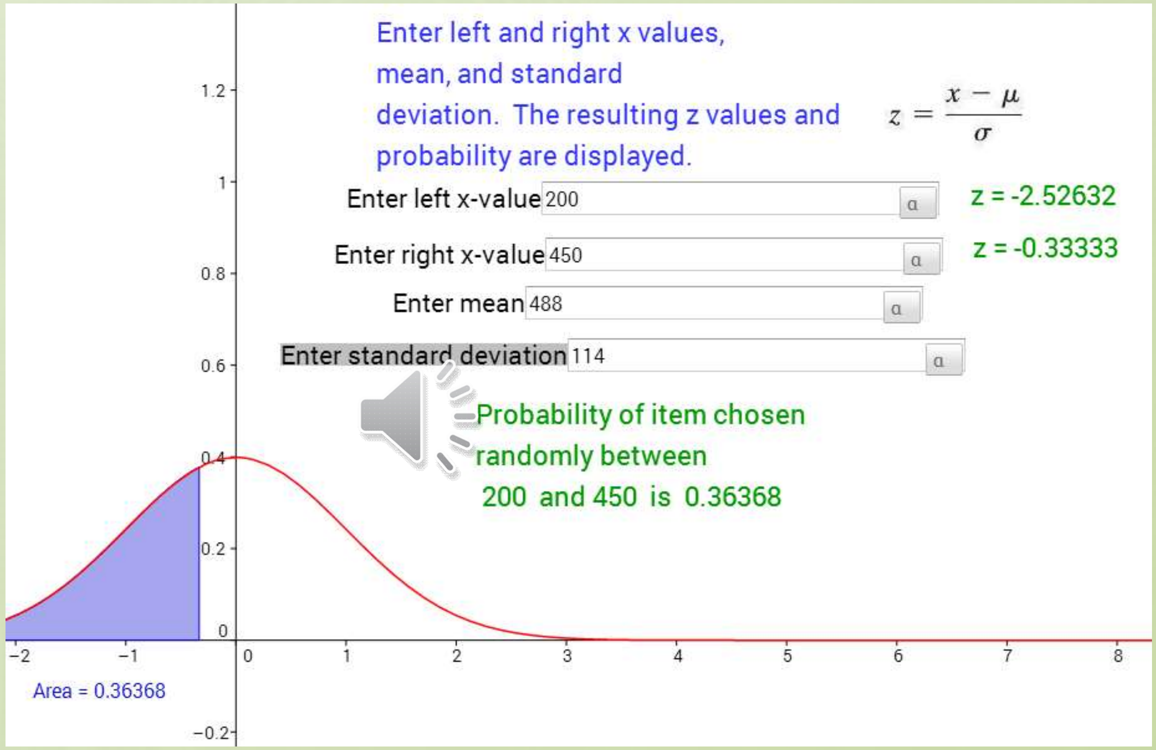


=NORM.DIST(C3,C4,C5,TRUE)-NORM.DIST(C2,C4,C5,TRUE)

B	C	D	E	F	G	H
left x =	200		z-score =	-2.52632		
right x =	450		z-score =	-0.33333		
mean =	488					
sd =	114					
cumulative						
probability =	0.36368					

More Technology

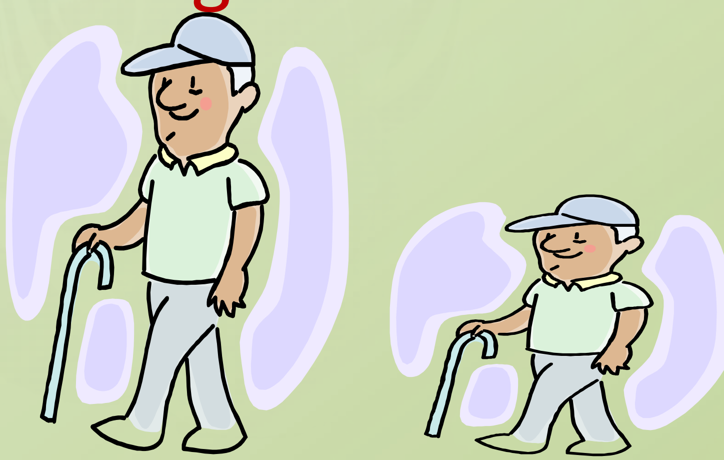
- Another way to do it



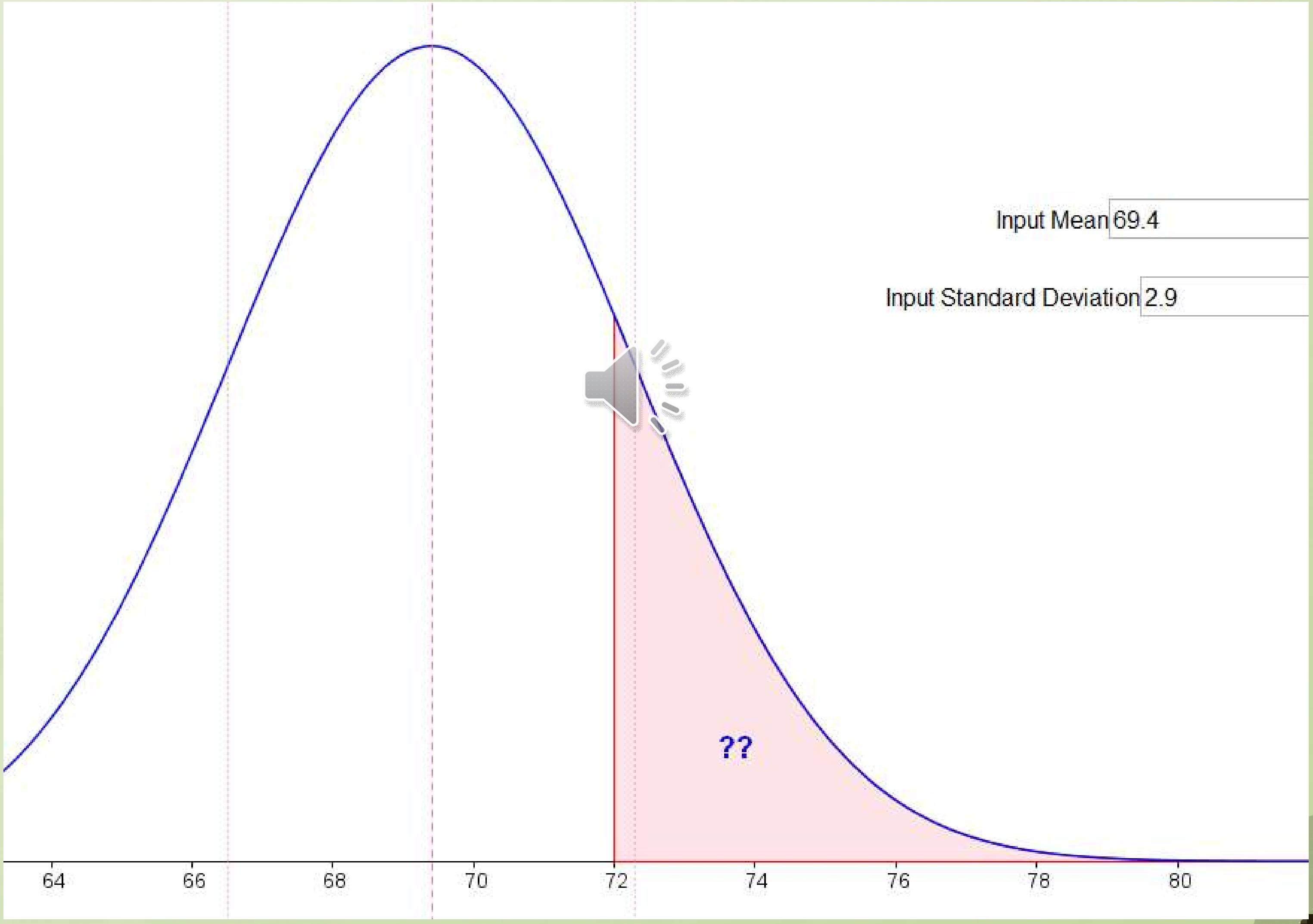
- https://www.geogebra.org/m/b6z3MetQ

What About to the Right?

- Given : In a survey of U.S. men, the heights in the 20 –29 age group were normally distributed, with a mean of 69.4 inches and a standard deviation of 2.9 inches. Find the probability that a randomly selected study participant has a height that is more than 72 inches



What About to the Right?



CONCEPTS AND | PRASE

What About to the Right?

- Remember … *total* area = 1
  - Calculate *left* area
  - Subtract from 1

- First, determine z-score

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{72 - 69.4}{2.9} = 0.8966$$

What About to the Right?

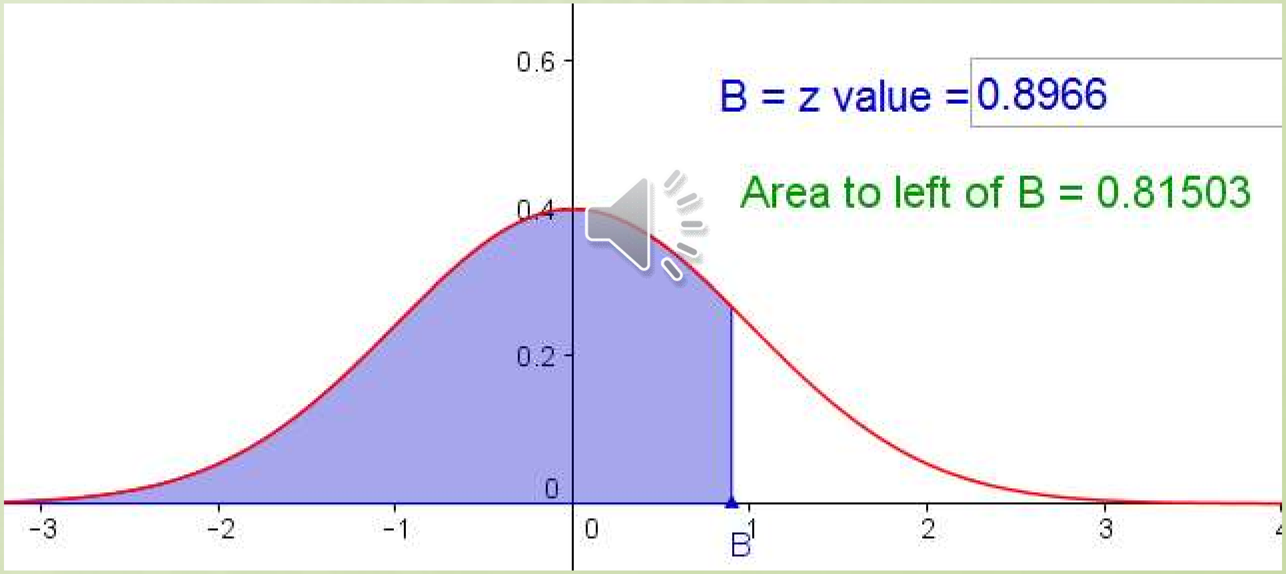
- Use Tables
  look up 0.9 (round up)
- Remember,
  this is the
  cumulative area
  to the *left*

z	.00
0.0	.5000
0.1	.5398
0.2	.5793
0.3	.6179
0.4	.6554
0.5	.6915
0.6	.7257
0.7	.7580
0.8	.7881
0.9	.8159
1.0	.8413

- Subtract from 1 to get area to *right*
  1 - 0.8159 = 0.1841

Use Technology

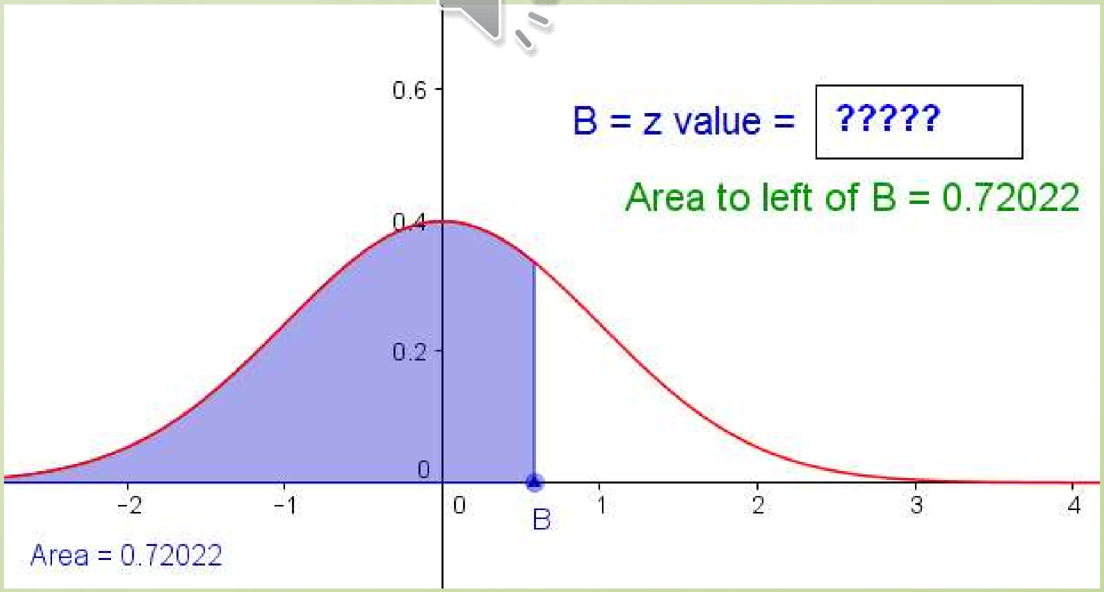
- Use app to determine



- Subtract 1 - 0.81503 = .18497

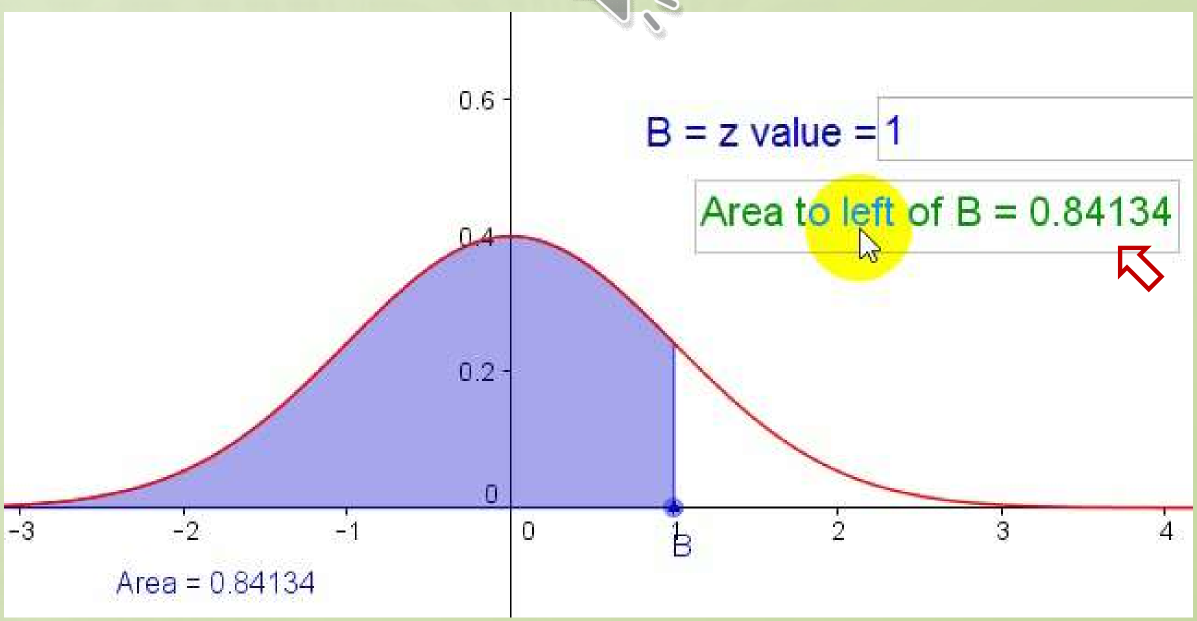
Going the Other Way

- What if we were given the *probability*
  - *That is the area under the curve (right or left)*
- Then asked to find the corresponding z-score



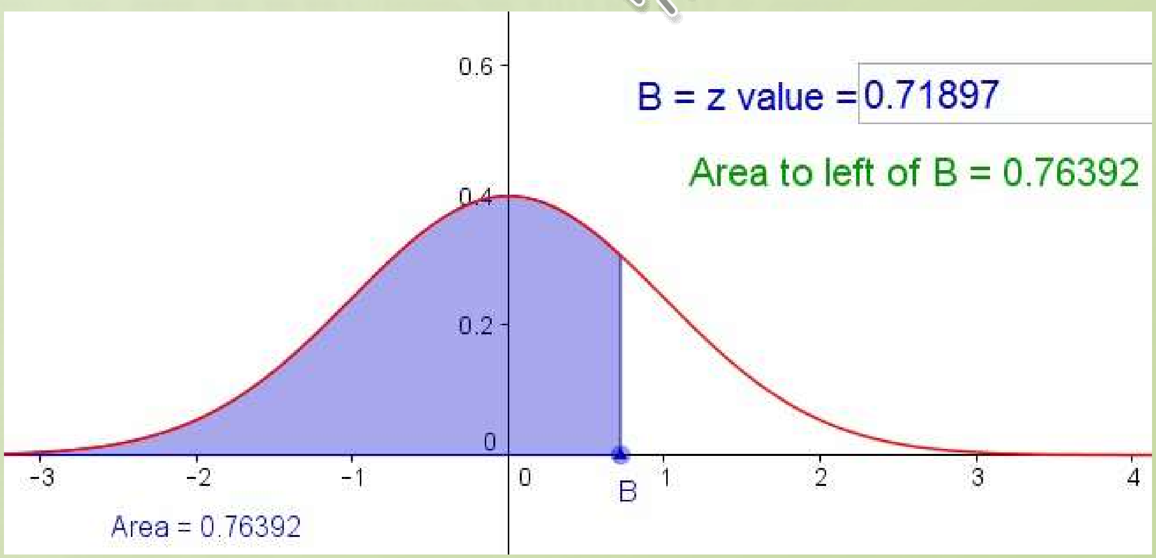
Going the Other Way

- We're looking for the z-score for the area to the left (the probability) of .72022
- We could manipulate the area to get the value and then note the z-score



Going the Other Way

- However … note that values for probability jump around
  - Might not be able to land on exact probability
- Try to find z-score for p = 0.75



Back to the Tables

- Now look in the *body* of tables

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

- Don’t see 0.7500?
  - Use closest value

Tables

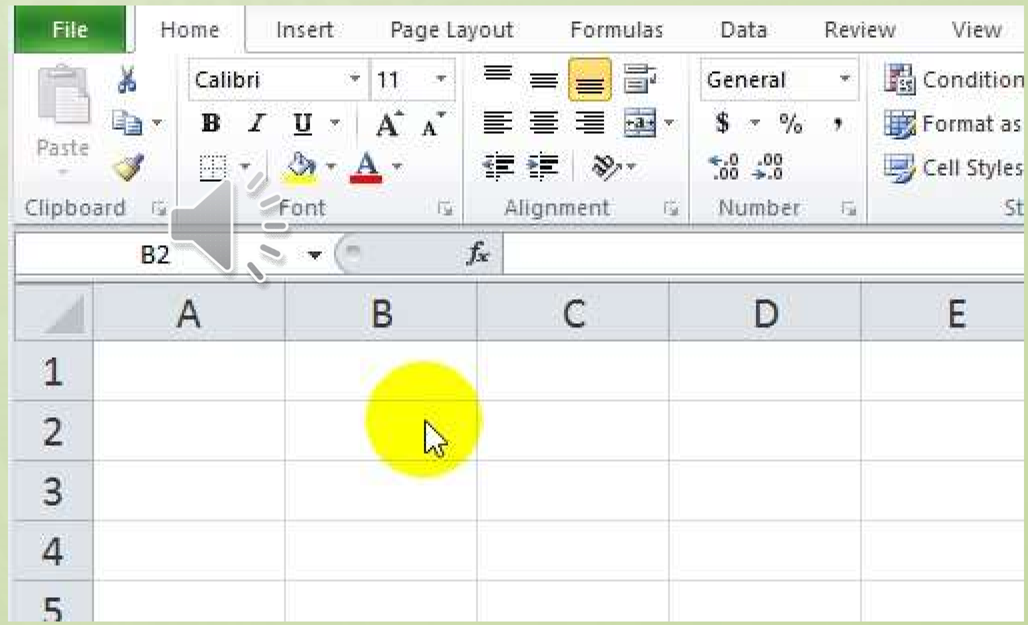
- We see 0.7486 is closest
- Look at row and column for z-score

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

- Z-score we use is $z = 0.67$

Find Z-Score with Excel

- Excel has a function which will find z-score value exactly



- Function is =NORM.S.INV*(probability value)*

Found the z … now find x

- From probability, we found z

- Use z to solve for x

- Also need mean and standard deviation

$$z = \frac{x - \mu}{\sigma}$$

$$z\sigma = x - \mu$$

$$\mu + z\sigma = x$$

$$x = \mu + z\sigma$$

Example

Try It Yourself 3

A veterinarian records the weights of dogs treated at a clinic. The weights are normally distributed, with a mean of 52 pounds and a standard deviation of 15 pounds. Find the weights x corresponding to z-scores of -2.33, 3.10, and 0.58. Interpret your results.

- Mean = 52
- Standard deviation = 15
- Now find x for given z-scores

$$z = \frac{x - \mu}{\sigma}$$

$$z\sigma = x - \mu$$

$$\mu + z\sigma = x$$

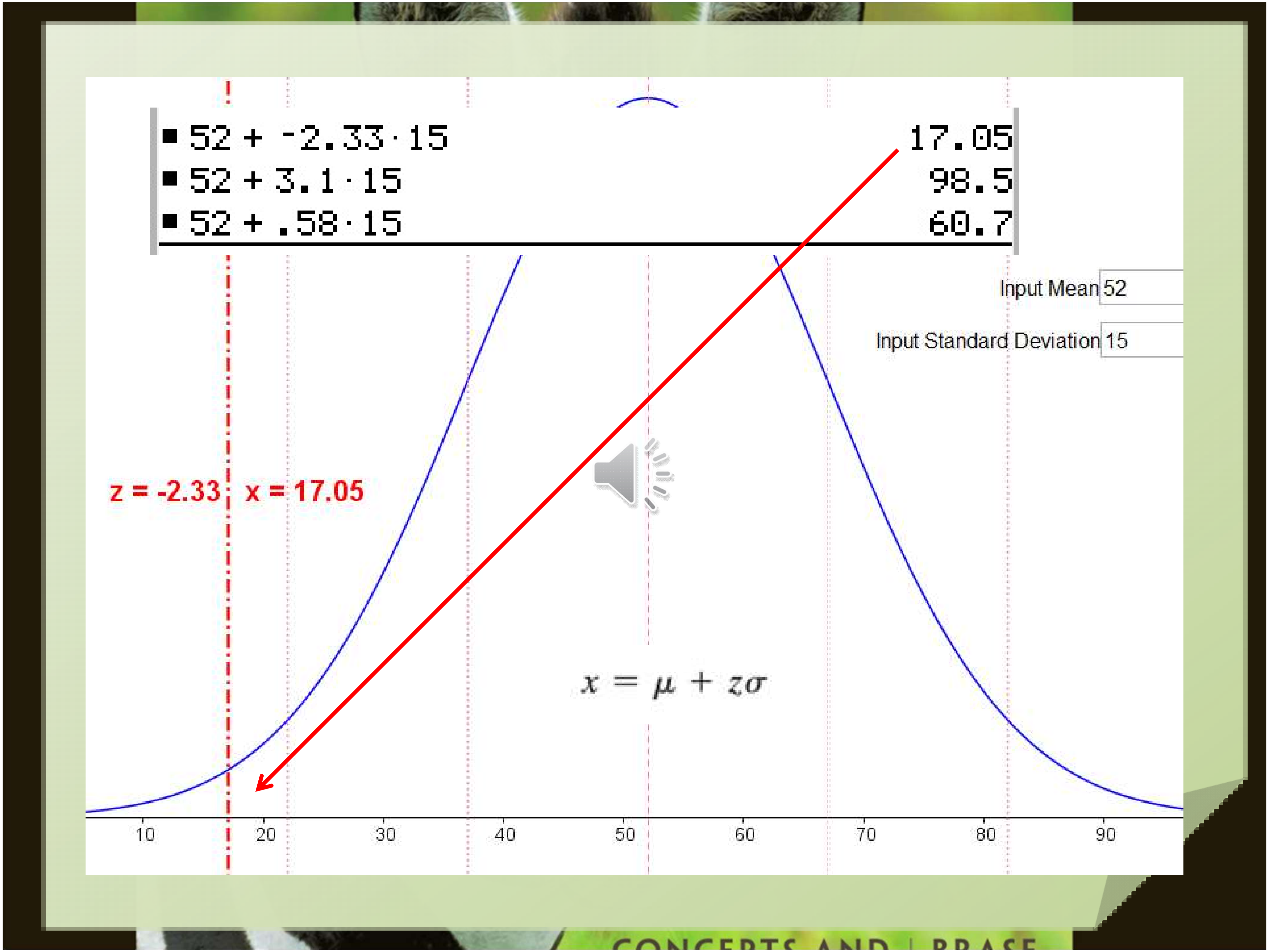
$$x = \mu + z\sigma$$

Example

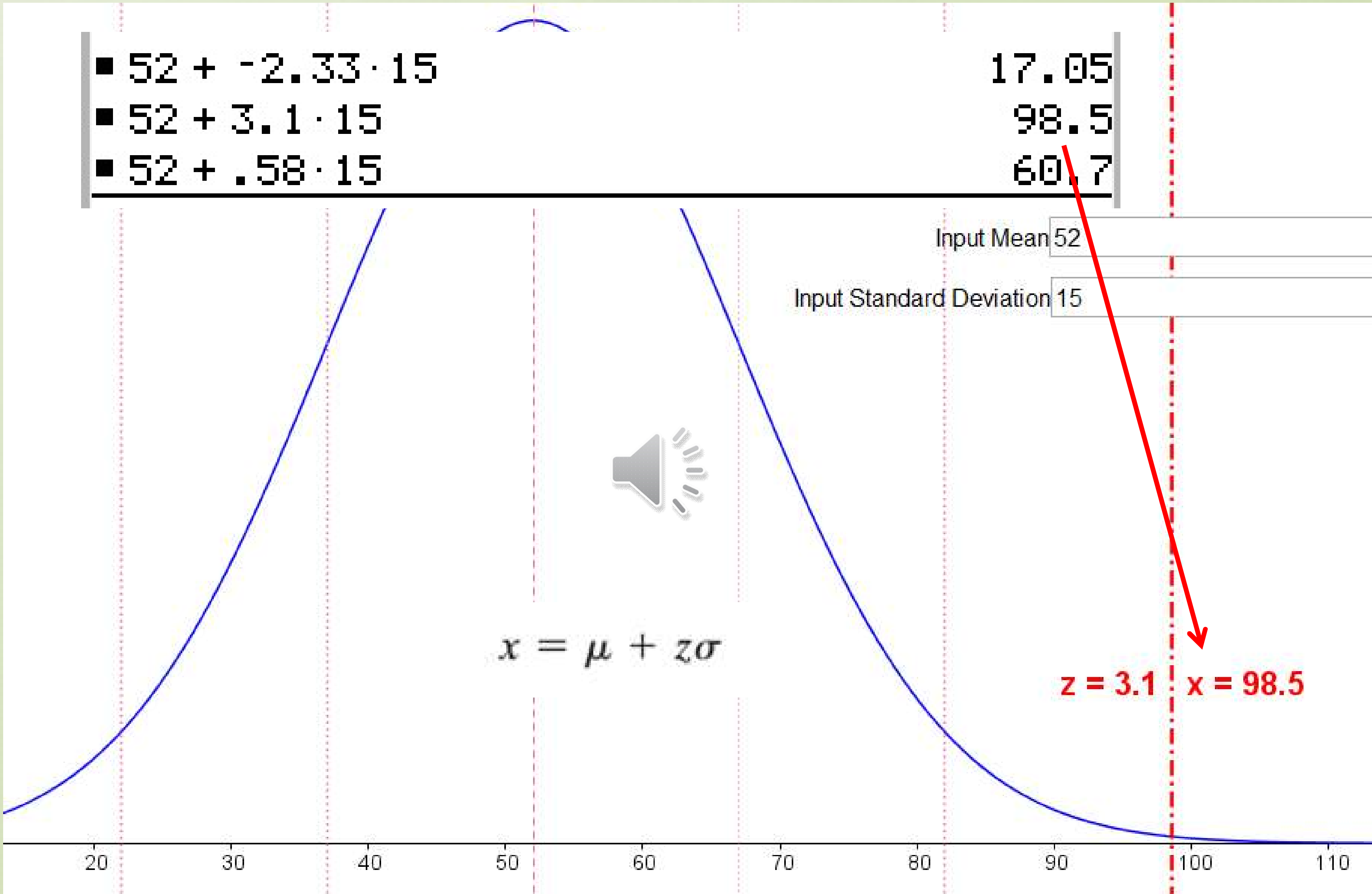
- Mean = 52
- Standard deviation = 15
- Now find x for given z-scores
  - z = -2.33
  - z = 3.1
  - z = .58

$$x = \mu + z\sigma$$

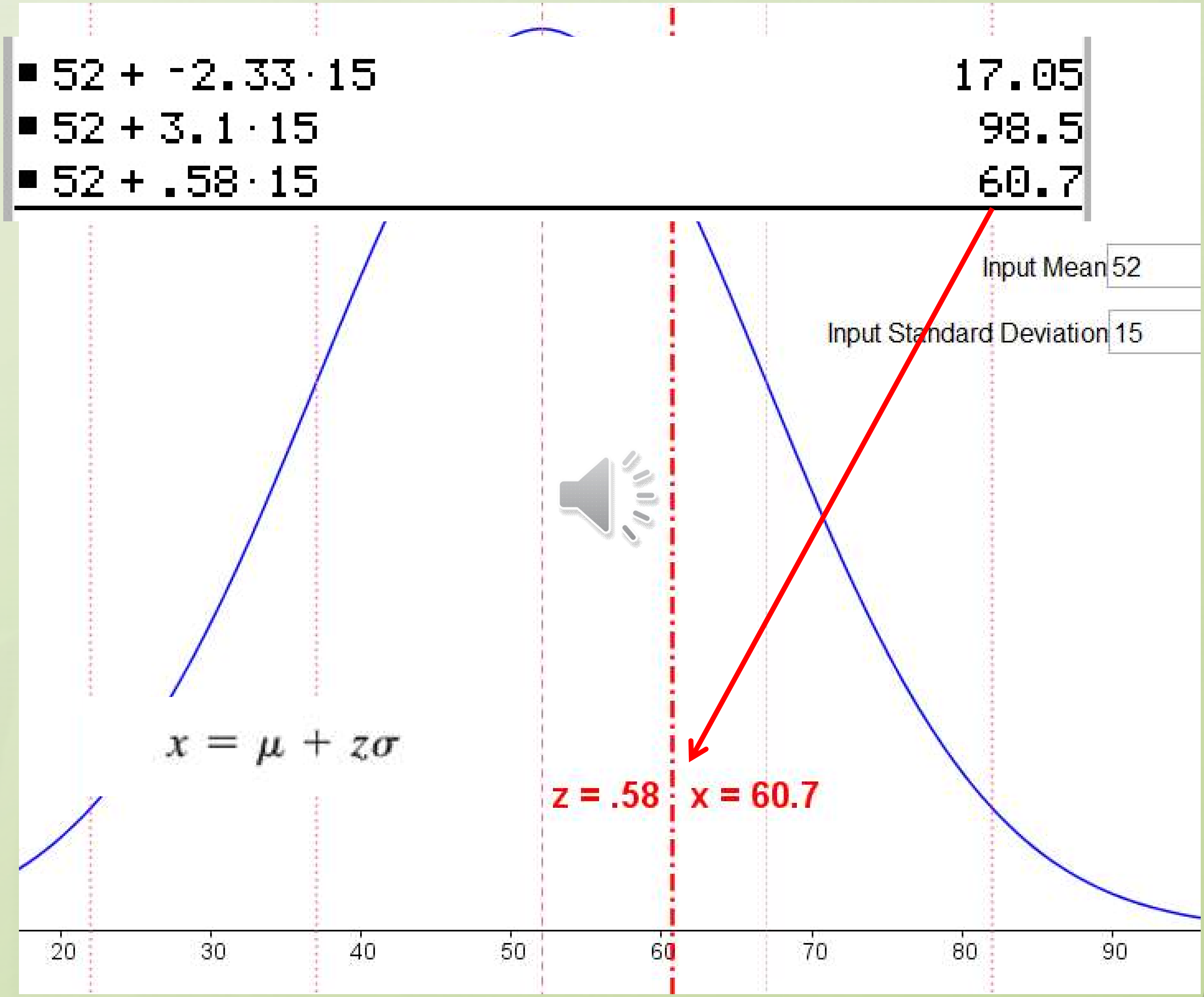
■ $52 + {}^{-}2.33 \cdot 15$	17.05
■ $52 + 3.1 \cdot 15$	98.5
■ $52 + .58 \cdot 15$	60.7



52 + -2.33 · 15 17.05
52 + 3.1 · 15 98.5
52 + .58 · 15 60.7
Input Mean 52
Input Standard Deviation 15
z = -2.33
x = 17.05
x = μ + zσ
10
20
30
40
50
60
70
80
90



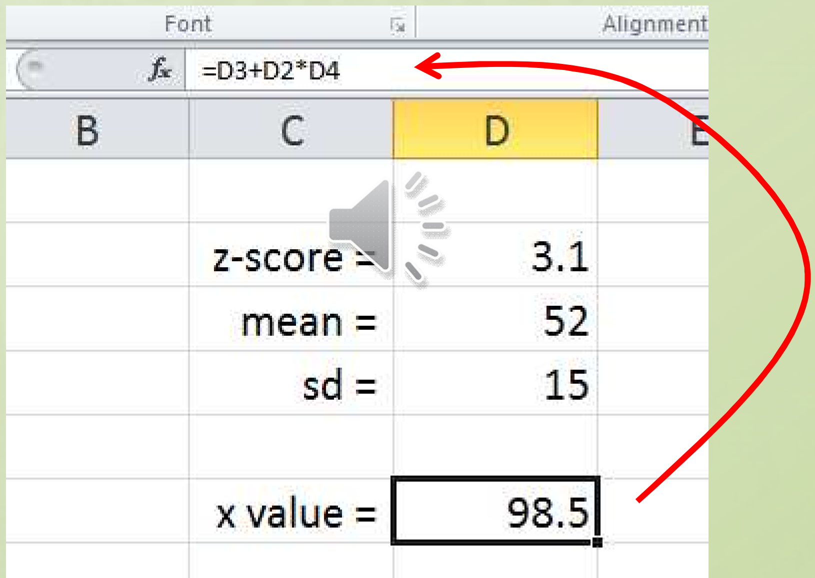
52 + -2.33 · 15 17.05
52 + 3.1 · 15 98.5
52 + .58 · 15 60.7
Input Mean 52
Input Standard Deviation 15
$x = \mu + z\sigma$
z = 3.1
x = 98.5
20
30
40
50
60
70
80
90
100
110



52 + -2.33·15 17.05
52 + 3.1·15 98.5
52 + .58·15 60.7
Input Mean 52
Input Standard Deviation 15
$x = \mu + z\sigma$
z = .58
x = 60.7
20
30
40
50
60
70
80
90

Use Technology

- An Excel Spreadsheet to calculate this:



- Use formula $x = \mu + z\sigma$

Given Probability, Find x

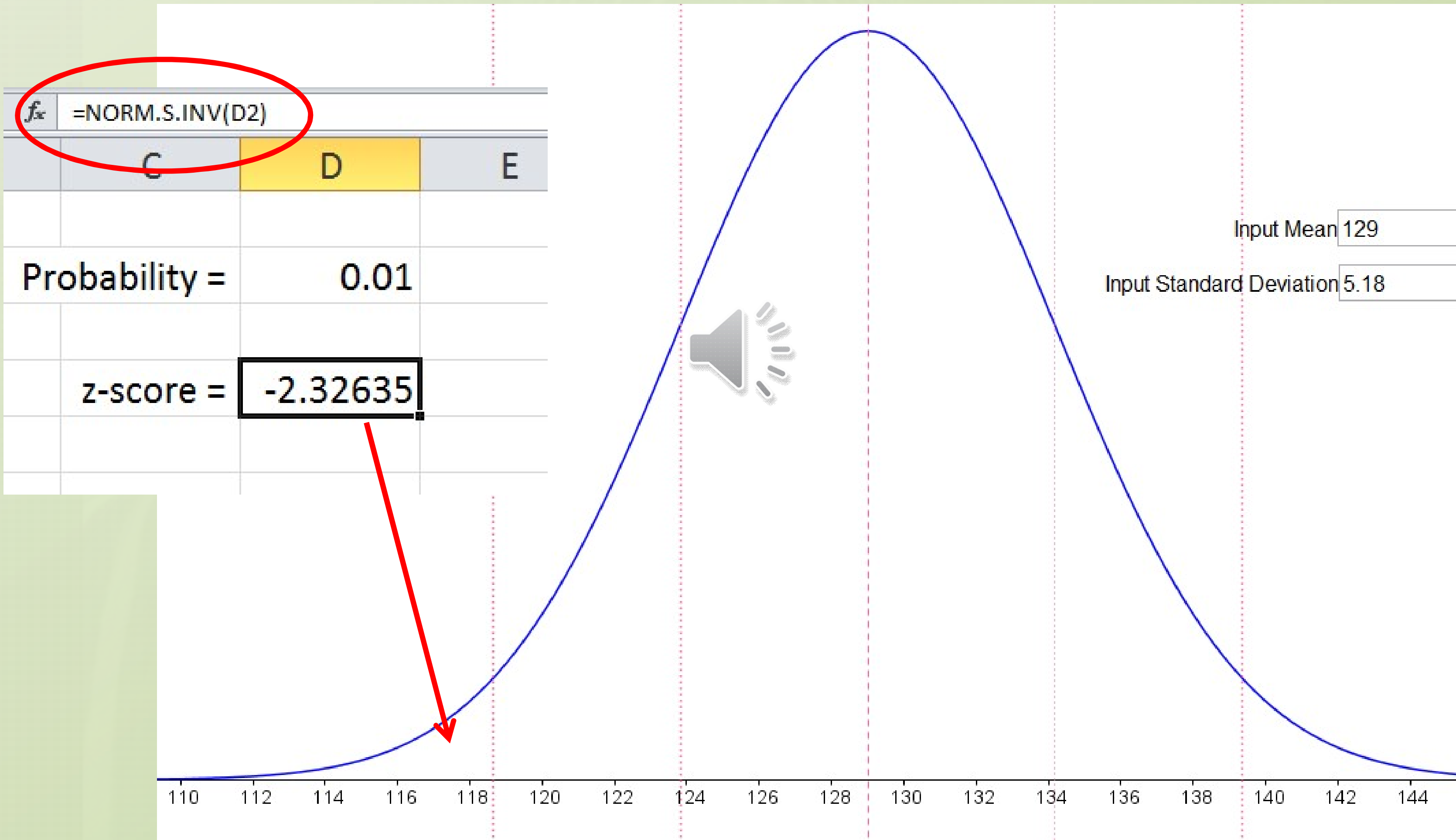
- Consider this problem

Try It Yourself 4

A researcher tests the braking distances of several cars. The braking distance from 60 miles per hour to a complete stop on dry pavement is measured in feet. The braking distances of a sample of cars are normally distributed, with a mean of 129 feet and a standard deviation of 5.18 feet. What is the longest braking distance one of these cars could have and still be in the bottom 1%? *(Adapted from Consumer Reports)*

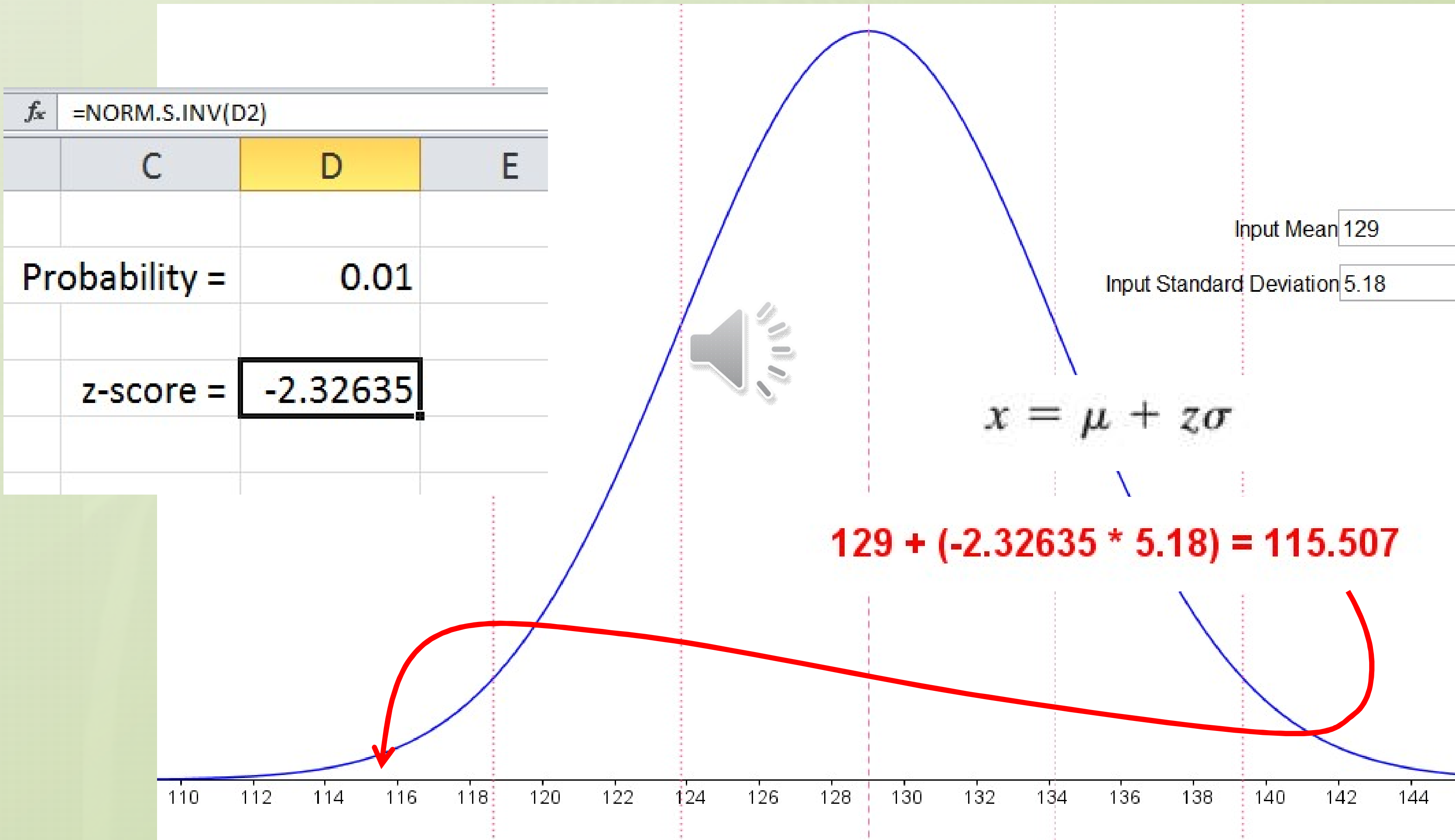
- Probability < 0.01

First, Find z



=NORM.S.INV(D2)
C
D
E
Probability =
0.01
z-score =
-2.32635
Input Mean 129
Input Standard Deviation 5.18
110
112
114
116
118
120
122
124
126
128
130
132
134
136
138
140
142
144

Now we have z, calculate x

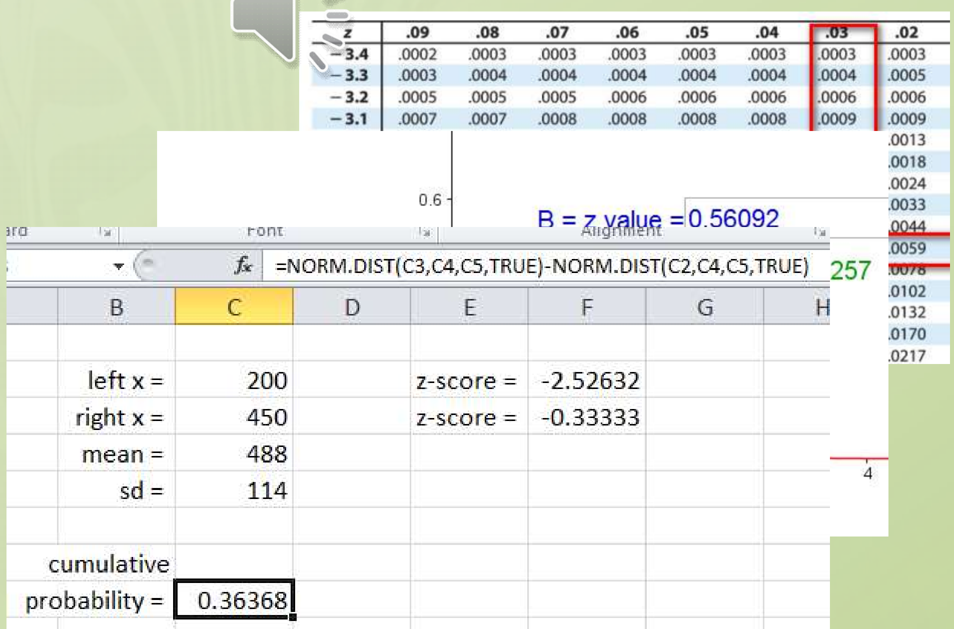


Summary

- Given x, mean, sd, find z

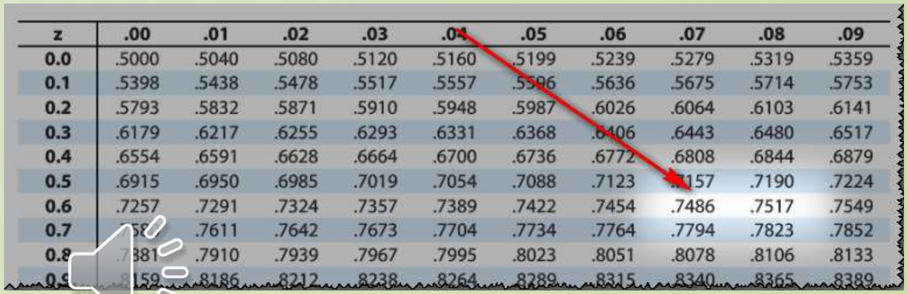
$$z = \frac{x - \mu}{\sigma}$$

- Given z, find probability … cumulative area under curve
  - Use tables
  - Use app
  - Use Excel



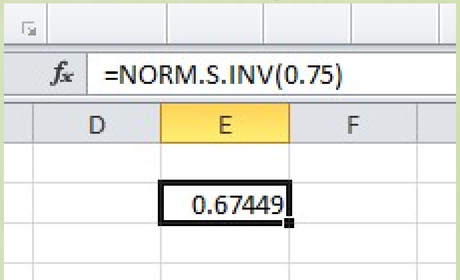
Summary

- Given probability, find z
  - Use tables



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	[illegible]	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

  - Use Excel



Summary

- Given probability, mean, sd … find x
- First use probability to determine z
  - App or Excel or tables "backwards"
- Then use z, mean, sd to find x

$$x = \mu + z\sigma$$

Using Normal Probability Distributions

Webinar Slides