Taxicab Geometry

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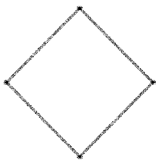
What Is It ???



Distance $d(P,Q) = |x_P - x_Q| = \sqrt{(x_P - x_Q)^2}$ $\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

Formula for measuring ⇔ metric

Axioms for metric space

- d(P, Q) ≥ 0 d(P, Q) = 0 ⇔ P = Q
- d(P, Q) = d(Q, P)
- d(P, Q) + d(Q, R) ≥ d(P, R)

Euclidian Distance Formula $d(P,Q) = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

- Does it satisfy all three axioms?

Consider this formula $d_T(P,Q) = |x_P - x_Q| + |y_P - y_Q|$

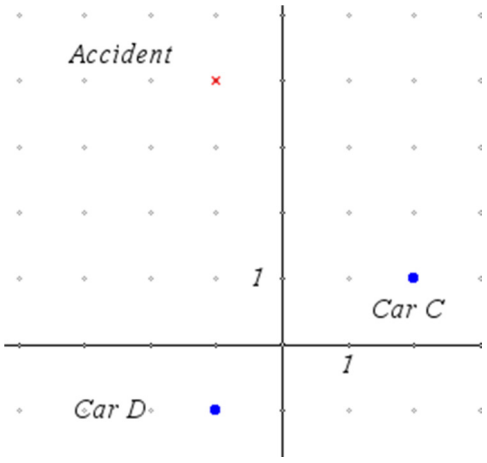
- Does it satisfy all three axioms?
- We call this formula the “taxicab” distance formula

Assumptions

- Model _____ geometry
- Streets “nice” _____
- No width streets
- Buildings “point mass”

Application of Taxicab Geometry
Accident at (-1,4).
Police Car C at (2,1) .
Police Car D at (-1,- 1).

Which car should be sent?



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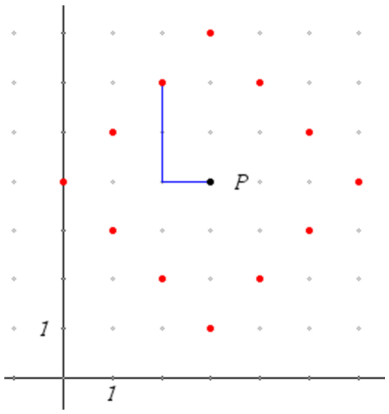
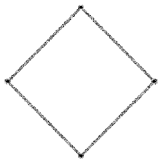
Circles $circle = \{P : d(P,C) = r, \quad r > 0, \quad C \ is \ fixed\}$



But … which metric?

Taxicab distance from P to each point?

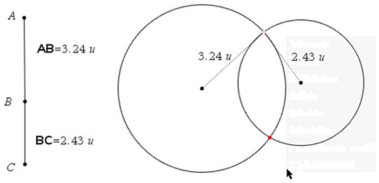
Again … What Is It ???



Taxicab Circle Construction on Nspire

1. Construct Euclidean circle with intersection points vertical, horizontal
2. Construct regular 4 sided polygon with vertices on intersection points
3. Hide the circle, vertical, horizontal lines

Ellipse $ellipse = \{P : d(P,F_1) + d(P,F_2) = d, \quad d > 0, \quad F_1,\ F_2 \ \ fixed\}$

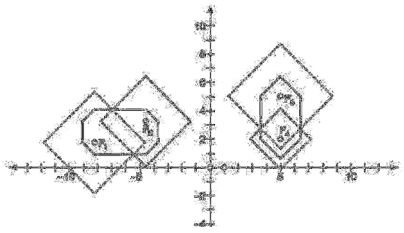


Special "slider"

- Divide line segment
- Transfer measurement of segments to circle radii
- Note circle intersection

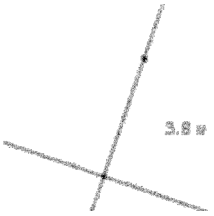
Taxicab Ellipse

- Same slider
- Note "circle" intersections
- Two possibilities



Point to Line Distance

- Shortest distance always on a perpendicular
- Also radius of circle tangent to the line



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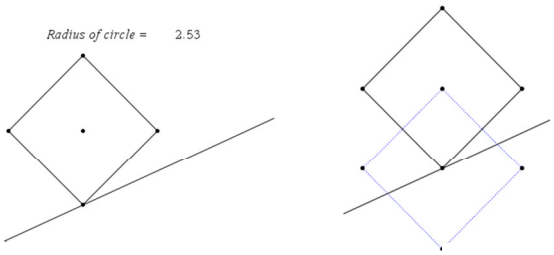
Taxicab Distance – Point to Line (or line to point)

Apply to taxicab circle

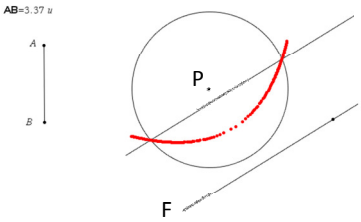
- When slope of line - 1 < m < 1 ?
- When slope, m = 1 ?
- When |m| > 1 ?
- Distance from line to point is not always ⊥ to line



Parabola

All points equidistant from a fixed point and a fixed line *(directrix)*

$$\{P : d(P,F) = d(P,k)\}$$

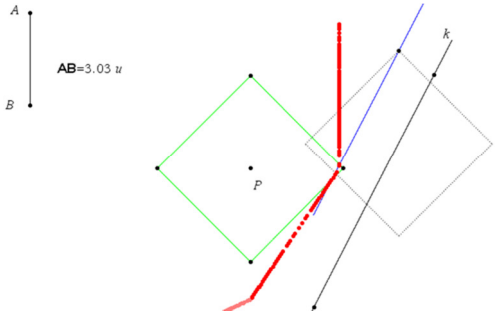


Taxicab Parabolas

From the definition

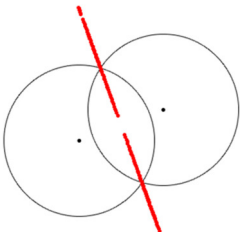
When directrix has slope m > 1

What does it take to have the "parabola" open downwards?

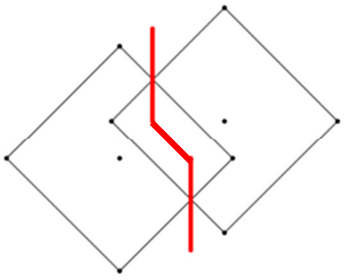


Locus of Points Equidistant from Two Points

Euclidean (perpendicular bisector)



Taxicab "perpendicular bisector"



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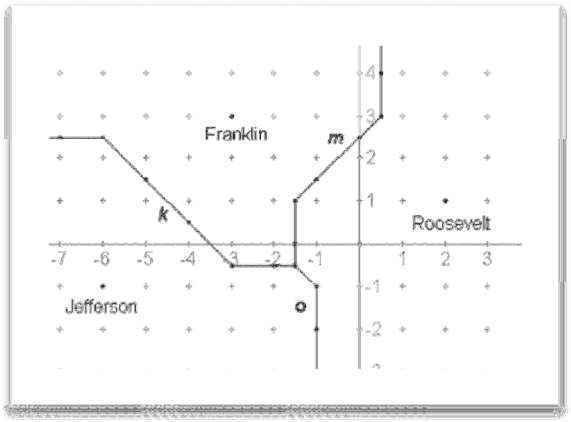
Application of Taxicab Geometry

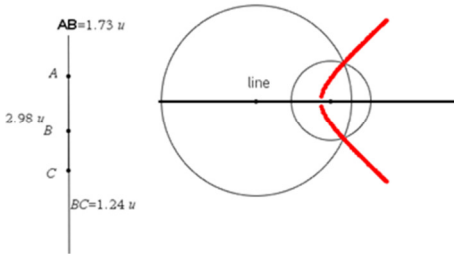
School district boundaries
Every student attends closest school.
Schools:
Jefferson at (-6, -1)
Franklin at (-3, -3)
Roosevelt at (2,1)
Find "lines" equidistant from each set of schools

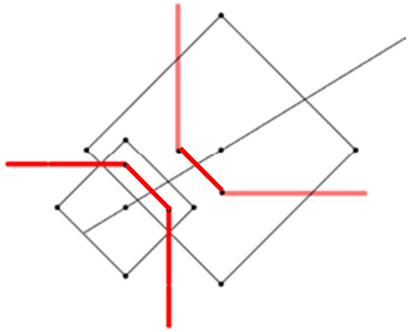


Hyperbola

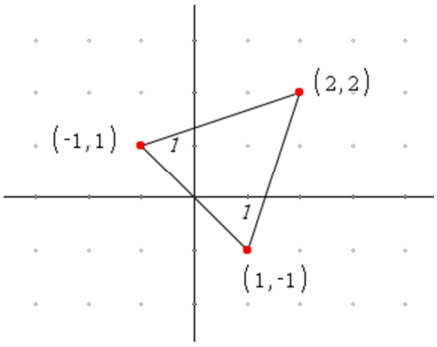
D(A, C) – D(B, C) = Constant = D(A, B)

Transfer lengths to circle radii





Taxicab Hyperbola



What is the taxicab length of the sides of this triangle?

How to classify the triangle?

Why?

- _____ create math
- Better understand Euclidian geometry
- Encourage _____
- Deeper appreciation of structure of math/geometry

Further Investigations

- _____ triangles
- Categories of _____
- Congruent triangles

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